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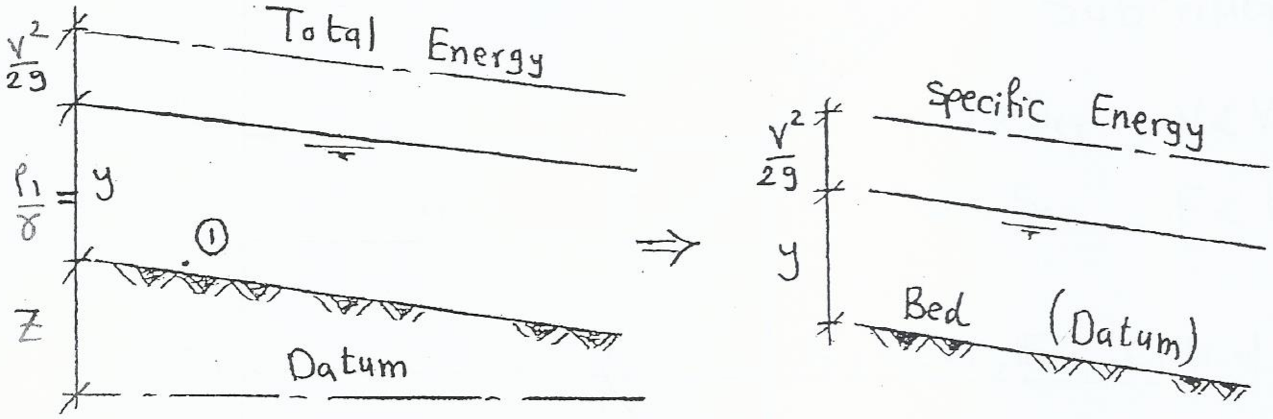
Specific Energy

Specific Energy



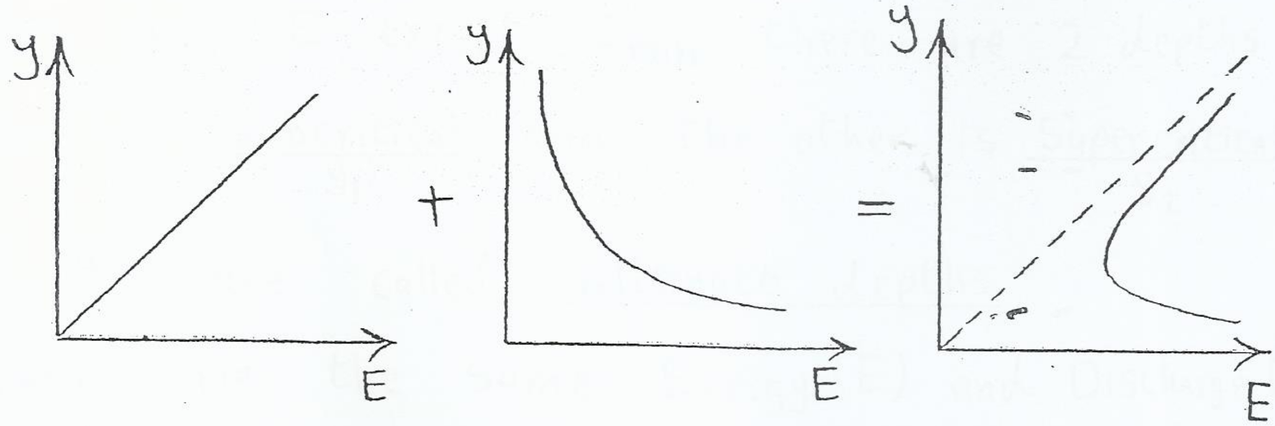
Energy (Bernoulli) Specific Energy

$$E = \frac{P_1}{\gamma} + \frac{V^2}{2g} + Z \Rightarrow E = y + \frac{V^2}{2g}$$

Definition

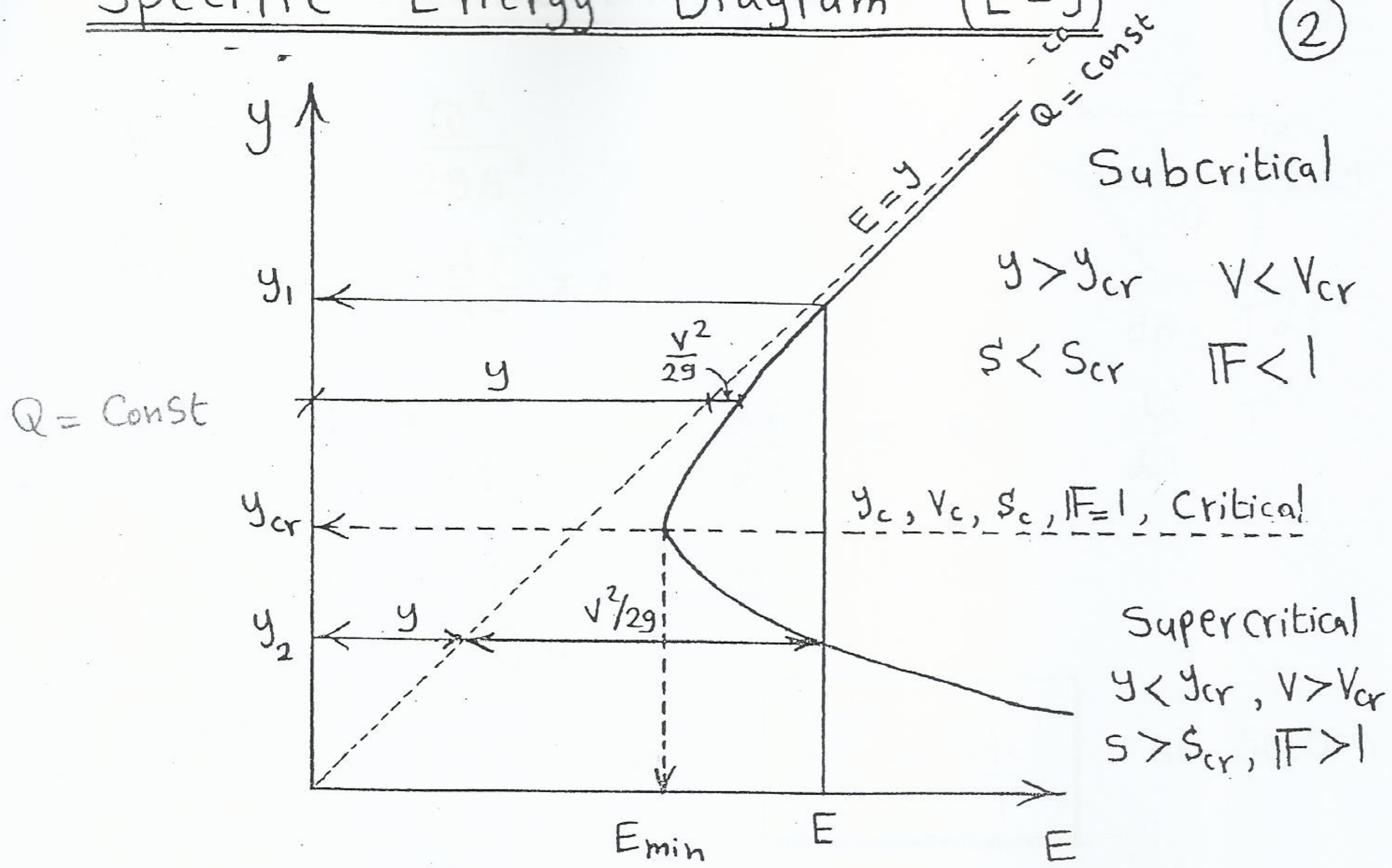
It is the energy referred to channel bed as a Datum.

E = Constant at any point along the channel



$E = y_1$ $E = \frac{V^2}{2g}$ $E = y + \frac{V^2}{2g}$

Specific Energy Diagram (E-y)



$y > y_{cr}$ ⇒ Subcritical flow

$y < y_{cr}$ ⇒ Supercritical flow

$y = y_{cr}$ ⇒ Critical flow at E_{min}

For any E except E_{min} there are 2 depths

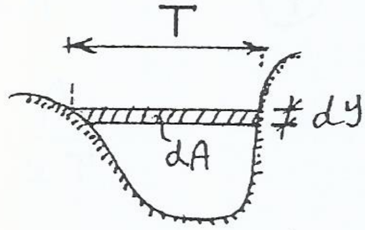
One is subcritical (y_1) and the other is Supercritical (y_2).

y_1, y_2 are called alternate depths.

They have the same Energy (E) and Discharge (Q)

Minimum Specific Energy (E_{min})

$$E = y + \frac{Q^2}{2gA^2}$$



$dA = T\,dy$

$\frac{dA}{dy} = T$

for E_{min} $\quad \frac{dE}{dy} = 0$

$$\frac{dE}{dy} = 1 + \frac{(-2)\,Q^2}{2g\,A_c^3}\left(\frac{dA}{dy}\right) = 0$$

$$= 1 - \frac{Q^2 T}{g\,A_c^3} = 0$$

$$\frac{Q^2 T}{g\,A_c^3} = 1 \quad \Rightarrow \quad \boxed{\frac{Q^2}{g} = \frac{A_c^3}{T}} \quad \text{any Section}$$

Rectangular Section

$$Q = qb \quad , \quad A_c = b\,y_{cr}$$

$$\frac{q^2 b^2}{g} = \frac{b^3 y_{cr}^3}{b}$$

$$y_{cr}^3 = \frac{q^2}{g}$$

$$\Rightarrow \quad \boxed{y_{cr} = \sqrt[3]{\frac{q^2}{g}}} \quad \text{Rec - Section only}$$

$$E = y + \frac{Q^2}{2gA^2} = y + \boxed{\frac{Q^2}{g}} \times \frac{1}{2A^2}$$ (4)

$$E_{min} = y_{cr} + \frac{A_c^3}{T} \times \frac{1}{2A_c^2} = y_{cr} + \frac{A_c}{2T}$$

$$\boxed{E_{min} = y_{cr} + \frac{1}{2} D_{cr}}$$, D = Hyd mean Depth $= \frac{A}{T}$

For Rectangular section $D = y_{cr}$

$$E_{min} = y_{cr} + \frac{1}{2} y_{cr}$$

$$\boxed{E_{min} = \frac{3}{2} y_{cr}}$$ Rec - Section only

$$\frac{1}{2} D_{cr} = \frac{V_{cr}^2}{2g} \Rightarrow V_{cr}^2 = g D_{cr}$$

$$\boxed{V_{cr} = \sqrt{g D_{cr}}}$$

$$\boxed{V_{cr} = \sqrt{g y_{cr}}}$$ Rec - Section only

Froude number

$$\mathbb{F}_N = \frac{V}{\sqrt{gD}}$$

$$\mathbb{F}_N = \frac{V}{\sqrt{gy}}$$ Rec - Section only

$\mathbb{F} > 1$ supercritical , $\mathbb{F} < 1$ Subcritical , $\mathbb{F} = 1$ critical

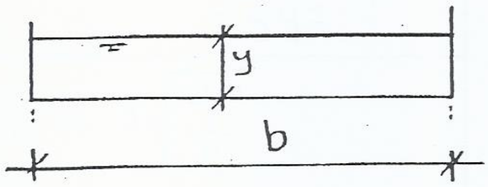
Great Width channel

$R = \frac{A}{P} \simeq y$

If $b > 10y$

$$R = \frac{A}{P} = \frac{by}{b+2y}$$

$$R = \frac{y}{1+\underbrace{2\frac{y}{b}}_{\simeq 0}} = y$$



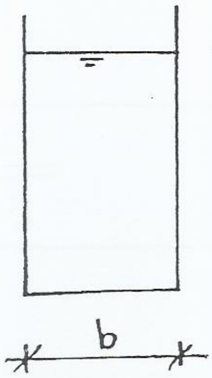
$\frac{y}{b} \simeq 0$

Deep channel

$R = \frac{b}{2}$

$$R = \frac{by}{b+2y} = \frac{b}{\underbrace{\frac{b}{y}}_{=0} + 2}$$

$$R = \frac{b}{2}$$



$\frac{b}{y} \simeq 0$

Very wide channel

$R = y$

$$Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

$$Q = \frac{1}{n} y^{2/3} S^{1/2} by$$

$$q = \frac{1}{n} S^{1/2} y^{5/3}$$

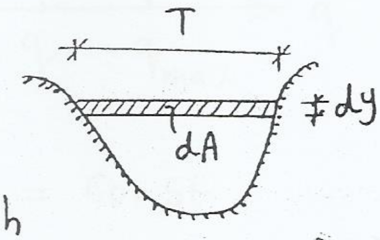
To get S_{cr} for very wide rect channel

$$q^2 = \frac{1}{n^2} S_{cr} y_c^{10/3}$$

$$g y_c^3 = \frac{S_{cr} y_{cr}^{10/3}}{n^2} \Rightarrow \boxed{S_{cr} = \frac{g n^2}{y_{cr}^{1/3}}}$$

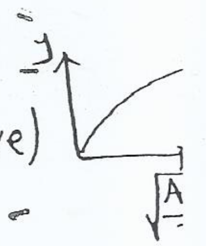
Any -channel	Rectangular channel
$E = y + \dfrac{Q^2}{2gA^2}$	$E = y + \dfrac{q^2}{2gy^2}$
$\dfrac{Q^2}{g} = \dfrac{A_c^3}{T}$	$y_c = \sqrt[3]{\dfrac{q^2}{g}}$
$E_{min} = y_{cr} + \dfrac{1}{2} D_{cr}$	$E_{min} = \dfrac{3}{2} y_c$
$V_c = \sqrt{g D_c}$	$V_c = \sqrt{g y_c}$
$\mathbb{F}_N = \dfrac{V}{\sqrt{gD}}$	$\mathbb{F}_N = \dfrac{V}{\sqrt{gy}}$
$D = \dfrac{A}{T}$	$y = \dfrac{A}{b}$
	$S_c = \dfrac{gn^2}{y_c^{1/3}}$ wide rect channel

$T = \dfrac{dA}{dy}$ = Top width



$D = \dfrac{A}{T}$ = Hydraulic mean depth

$M = \sqrt{\dfrac{Q^2}{g}} = \sqrt{\dfrac{A_c^3}{T}}$ = Section factor (M curve)



for non rect channels y_c لإيجاد (M curve) يستخدم

$S_c = \dfrac{gn^2}{y_c^{1/3}}$ very wide rectangular channel

Specific Discharge Diagram $(q-y)$ curve

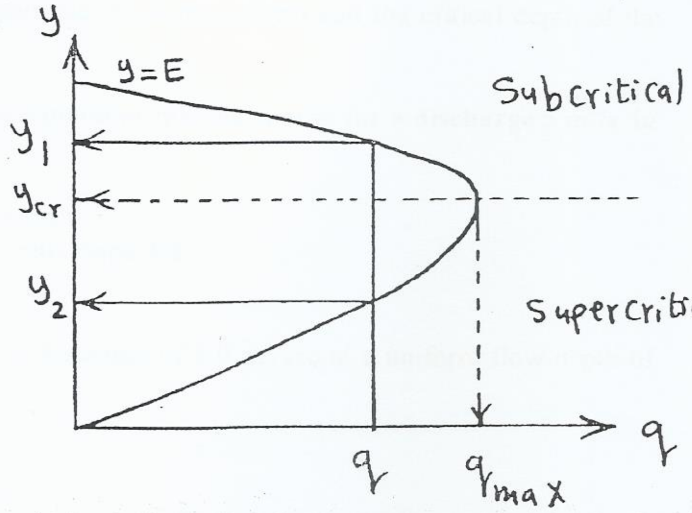
$$E = y + \frac{Q^2}{2gA^2}$$

$$E - y = \frac{Q^2}{2gA^2}$$

$$\boxed{Q = A\sqrt{2g(E-y)}}$$

$$\boxed{q = y\sqrt{2g(E-y)}}$$

Rec - section only



q_{max} at y_{cr}

$$y_{cr} = \frac{2}{3}E$$

$$q_{max} = \sqrt{g\, y_{cr}^3}$$

$$b_{min} = \frac{Q}{q_{max}}$$

E = Const

ASSIGNMENT NO. 5
Specific Energy

1. Define each of the following:-
   a) Specific energy
   b) Alternate depths

2. Sketch each of the following curves and discuss each of them
   a) specific energy curve
   b) Specific discharge diagram

3. **A flow of 5.0 m^3/s is passing at a depth of 1.5 m through a rectangular channel of width 2.5 m. What is the value of the depth alternate to the existing depth?**

4. A rectangular channel 2.5 m wide has a specific energy of 1.5m when carrying a discharge of 6.48 m^3/s. Calculate the alternate depths and corresponding Froude **numbers**.

5. A 50 ft wide rectangular channel has slope S = 0.0025 and Manning's coefficient n = 0.035 is carrying a discharge of 2200 ft^3/sec. Determine the normal depth and the critical depth of the flow. Construct the specific energy curve.

6. **Calculate the critical depth and the corresponding specific energy for a discharge 5 m^3/s in the following channels:**
   a) Rectangular channel B= 2.0 m
   b) Triangular channel side slope 2:1
   c) Trapezoidal channel , B= 2.0 m side slope 3:2
   d) Circular channel D = 2.0 m

7. A rectangular channel 5 m wide conveys a discharge of 8.0 m^3/sec at a uniform flow depth of 1.25m.
   Determine the following:
   a) the critical depth
   b) the minimum energy
   c) the critical velocity
   d) the flow conditions
   e) Froude's number at the critical depth and at depth 1.25 m
   f) The critical slope

8. **Twenty two cubic meters per second flow in a rectangular channel of 6 m width having n of 0.017. Plot accurately the specific energy diagram for depths from 0 to 3 m using the same scale for y and E. Determine from the diagram,**
   (a) The critical depth.
   (b) The minimum specific Energy.
   (c) The specific energy when the depth of flow is 2 m.
   (d) The depths when the specific energy is 2.5 m.
   (e) The depth which is the alternate depth for 1.5 m depth.
   (f) What type of flow exists when the depth is: (i) 0.6 m. (ii) 1.8 m.

(g) What are the channel slopes necessary to maintain these depths?
(h) What types of slopes are these?
(k) What is the critical slope assuming the channel to be of great width?

9. Flow occurs in rectangular channel of 20 ft width and has a specific energy of 10 ft. Plot accurately the q-curve and determine the following from the curve:
   a) the critical depth and maximum flow rate
   b) the flow rate at a depth of 8 ft.
   c) the depths at which a flow rate of 1000 cfs may exist
   d) the flow condition at these depths

10. **A storm culvert 1.2 m diameter is flowing half full and the flow is in critical state. Estimate the discharge and the specific energy.**

11. A 12.5-m wide rectangular channel carries a 32 m^3/s at a depth of 2 m. Is this flow subcritical or supercritical? If n = 0.025, what is the critical slope of this channel for this discharge? What channel slope must be provided to produce a uniform flow at the depth of 2 m?

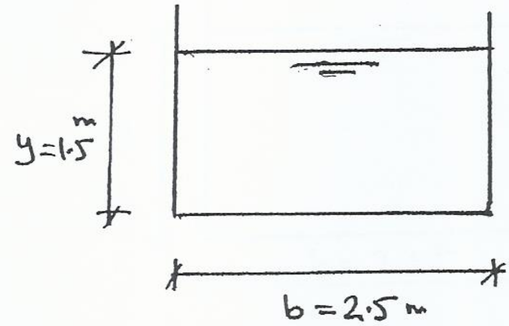
12. **For a constant specific energy of 1.5 m, what is the maximum flow that may occur in a rectangular channel of 5.0 m wide? Draw the Q-y curve for this section.**

Question (3) :-

$Q = 5 \; m^3/s$

$y_1 = 1.5 \; m$

$b = 2.5 \; m$



$$q = \frac{Q}{b} = \frac{5}{2.5} = 2 \; m^3/s/m'$$

$$E = y_1 + \frac{q^2}{2gy_1^2} = y_2 + \frac{q^2}{2gy_2^2}$$

$$\Rightarrow \; 1.5 + \frac{(2)^2}{2 * 9.81 (1.5)^2} = y_2 + \frac{(2)^2}{2 * 9.81 \, y_2^2}$$

$$1.59 = y_2 + \frac{1}{4.9 y_2^2}$$

$$7.79 y_2^2 = 4.9 y_2^3 + 1$$

$$4.9 y_2^3 - 7.79 y_2^2 + 1 = 0$$

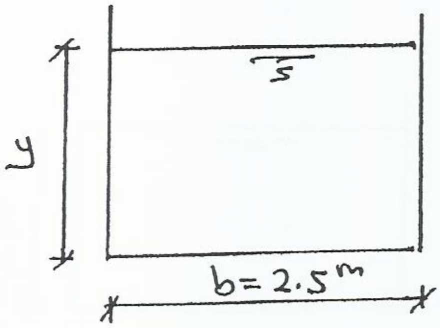
by trials & error $\Rightarrow y_2 = 0.42 \, m$

uestion (4) :-

$b = 2.5$ m

$E = 1.5$ m

$Q = 6.48\ m^3/s$



Req y_1, y_2 & F_{N_1}, F_{N_2}

Sol.

$$q = \frac{Q}{b} = \frac{6.48}{2.5} = 2.59\ m^3/s/m'$$

$$E = y + \frac{q^2}{2gy^2}$$

$$1.5 = y + \frac{(2.592)^2}{2*9.81 y^2}$$

$$292 y^3 - 4.39 y^2 + 1 = 0$$

$y_1 = 0.62$ m

$y_2 = 1.3$ m

$$V_1 = \frac{q}{y_1} = \frac{2.592}{0.62} = 4.18\ m/s$$

$$V_2 = \frac{q}{y_2} = \frac{2.592}{1.3} = 1.99\ m/s$$

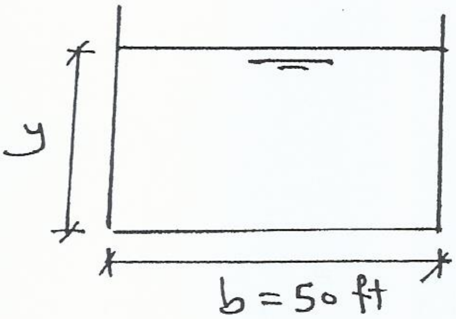
$$F_{N_1} = \frac{V_1}{\sqrt{gy_1}} = \frac{4.18}{\sqrt{9.81 * 0.62}} = 1.69 > 1 \quad \text{Super Critical}$$

$$F_{N_2} = \frac{V_2}{\sqrt{gy_2}} = \frac{1.99}{\sqrt{9.81 * 1.3}} = 0.55 < 1 \quad \text{Sub Critical}$$

Question (5) :-

$S = 0.0025$ & $n = 0.035$

$Q = 2200 \ ft^3/s$



Req * $y = y_n$

* y_{cr}

* (E − y) Curve

Sol.

$$Q = \frac{1.49}{n} R^{2/3} S^{\frac{1}{2}} A$$

$$2200 = \frac{1.49}{0.035} \left(\frac{50y}{50+2y}\right)^{2/3} (0.0025)^{\frac{1}{2}} (50y)$$

$$20.65 = \left(\frac{50y}{50+2y}\right)^{2/3} y$$

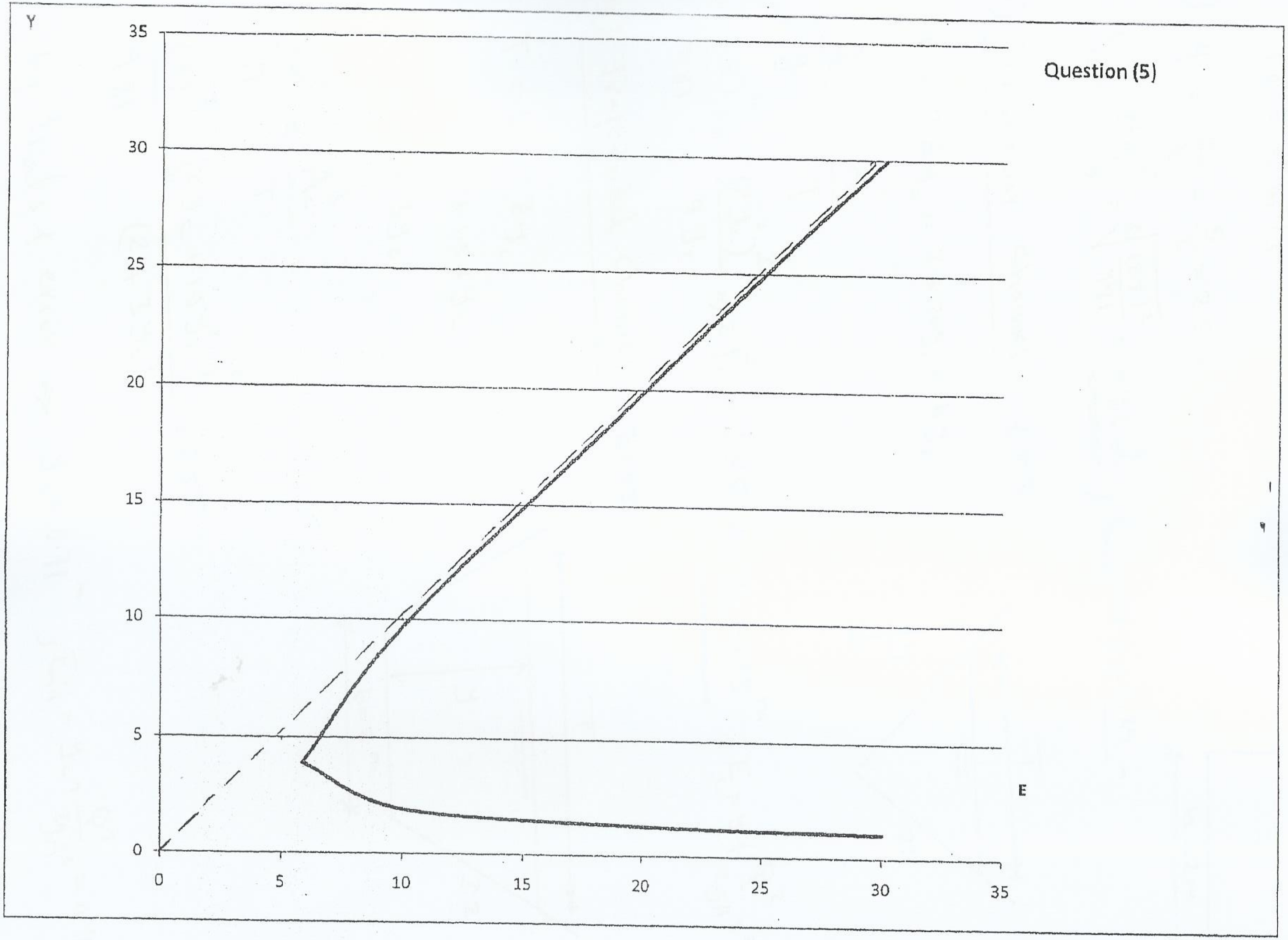
by trials & error $\Rightarrow$ $y = 6.15$ ft

$$* \quad q = \frac{Q}{b} = \frac{2200}{50} = 44 \quad ft^3/s/ft$$

$$\therefore y_{cr} = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(44)^2}{32.2}} = 3.92 \ ft$$

$$E = y + \frac{q^2}{2gy^2} = y + \frac{(44)^2}{2 * 32.2 y^2} = y + \frac{30}{y^2}$$

y	2	3	3.92	5	10	15	20	25	30	35
E	9.5	6.3	5.88	6.2	10.3	15.1	20.07	25.05	30.03	35.02



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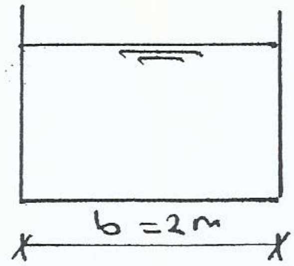
Question (6) :-

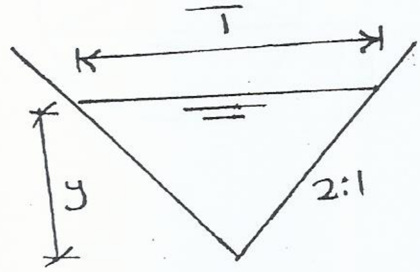
$Q = 5\ m^3/s$



a) $q = \frac{Q}{b} = \frac{5}{2} = 2.5\ m^3/s/m'$

$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(2.5)^2}{9.81}} = 0.86\ m$, $E_{min} = 1.5 y_c = 1.29\ m$

b) Triangular channel $\quad Z = 2$

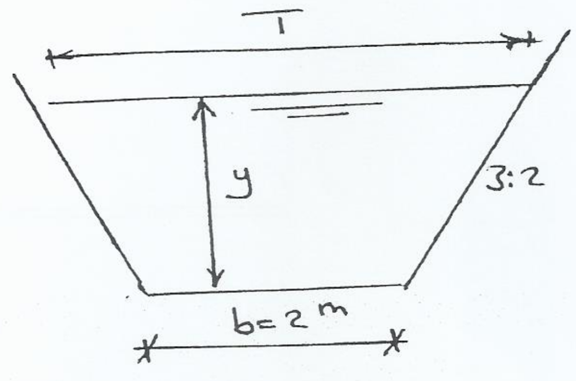


$T = 2Zy_c = 2 * 2 * y_c = 4y_c$

$\frac{Q^2}{g} = \frac{A_c^3}{T}$

$\frac{(5)^2}{9.81} = \frac{(2y_c^2)^3}{4y_c} \Rightarrow 2y_c^5 = 2.55 \Rightarrow y_c = 1.05^m$, $E_c = y_c + \frac{Q^2}{2gA^2} = 1.31^m$

c) Trapezoidal channel $\quad Z = 1.5$



$T = b + 2Zy_c$

$= 2 + 2 * 1.5 * y_c$

$= 2 + 3y_c$

$\frac{Q^2}{g} = \frac{A_c^3}{T}$

$\frac{(5)^2}{9.81} = \frac{(2y_c + 1.5y_c^2)^3}{(2 + 3y_c)} = 2.55$

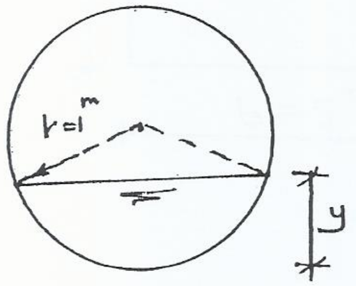
by trials + error $\Rightarrow y_c = 0.71^m$, $E_{min} = y_c + \frac{Q^2}{2gA^2} = 0.98^m$

d) Circular channel :-

$$A_c = \frac{1}{2} r^2 (\theta - \sin\theta)$$

$$= \frac{1}{2}(1)^2 (\theta - \sin\theta)$$

$$\therefore A_c = 0.5(\theta - \sin\theta)$$



$$T = 2r \sin\frac{\theta}{2} = 2\sin\frac{\theta}{2}$$

$$\frac{Q^2}{g} = \frac{A_c^3}{T}$$

$$2.55 = \frac{\left[0.5(\theta - \sin\theta)\right]^3}{\left(2\sin\frac{\theta}{2}\right)}$$

by trial & error $\Rightarrow \theta = 3.29$ rad $\cong 188.5^\circ$

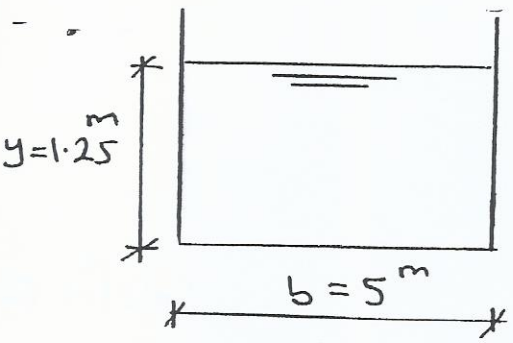
$$\therefore y_c = r - r\cos\frac{\theta}{2} = 1 - 1\cos\frac{188.5}{2} = 1.074^m$$

$$E_{min} = y_c + \frac{Q^2}{2gA_c^2} = 1.16 \text{ m}$$

Question (7) :-

$Q = 8 \ m^3/s$

$y = 1.25 \ m$



a) $q = \frac{Q}{b} = \frac{8}{5} = 1.6 \ m^3/s/m$

$y_{cr} = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(1.6)^2}{9.81}} = 0.64 \ m$

b) $E_{min} = 1.5 y_c = 1.5 * 0.64 = 0.96 \ m$

c) $V_c = \sqrt{g y_c} = \sqrt{9.81 * 0.64} = 2.5 \ m/s$

d) $y > y_{cr} \Rightarrow$ subcritical flow

e) $F_N = \frac{V}{\sqrt{gy}} = \frac{(Q/by)}{\sqrt{gy}} = \frac{\left(\frac{8}{5*1.25}\right)}{\sqrt{9.81*1.25}} = 0.36 < 1$ (subcri

f) $Q = \frac{1}{n} R_{cr}^{2/3} S_{cr}^{1/2} A_{cr}$ assume $n = 0.025$

where $A_{cr} = 5 * 0.64 = 3.2 \ m^2$

$P_{cr} = 5 + 2*0.64 = 6.28 \ m$

$R_{cr} = \frac{A_{cr}}{P_{cr}} = \frac{3.2}{6.28} = 0.51 \ m$

For Critical Condition

$\therefore 8 = \frac{1}{0.025} (0.5)^{2/3} (S_{cr})^{1/2} (3.2) \Rightarrow S_{cr} = 0.0098$

$S_{cr} = 9.84 \ m/km$

Question (8) :-

$Q = 22 \ m^3/s$

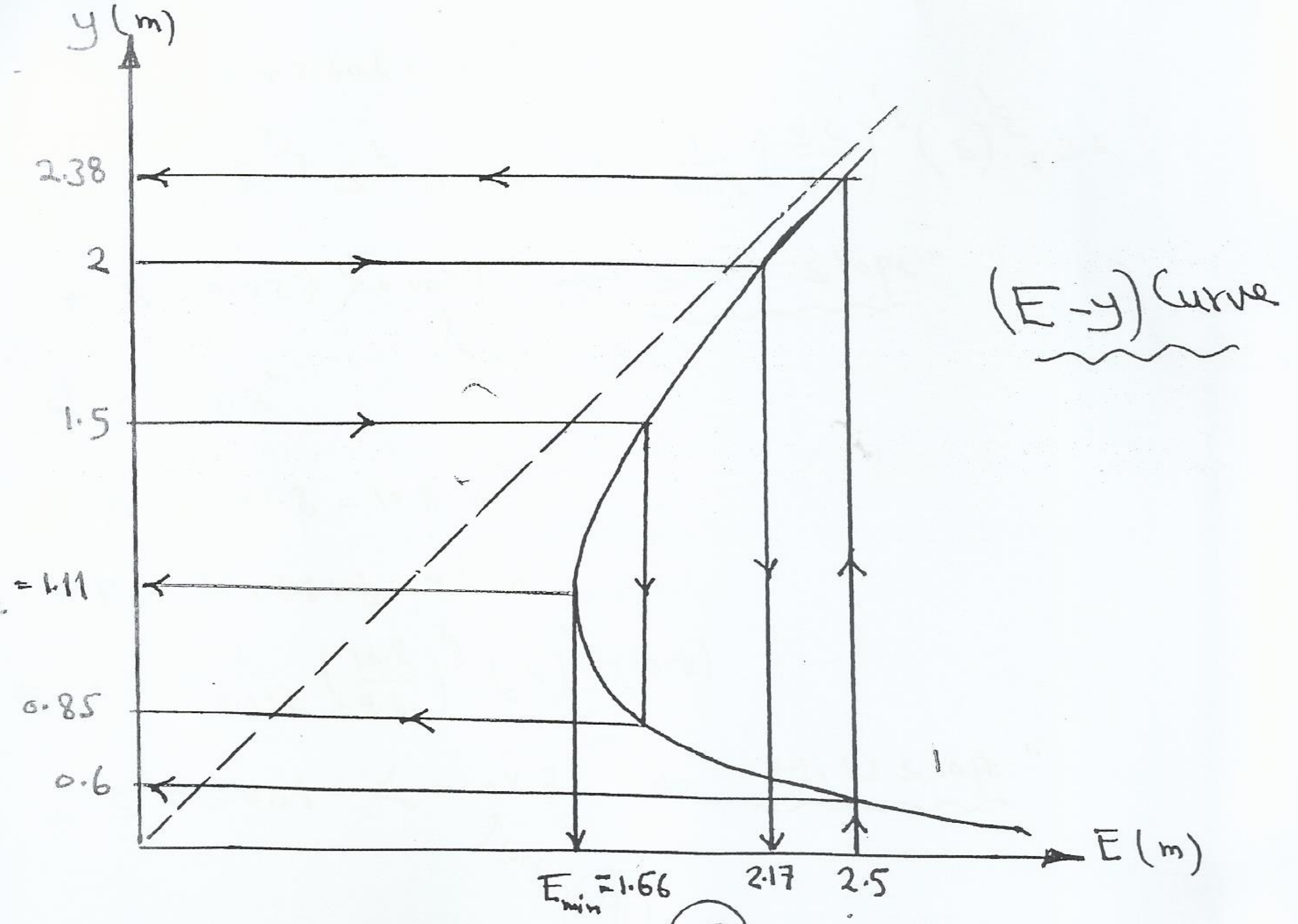
$n = 0.017$

$E = y + \dfrac{q^2}{2gy^2}$, $q = \dfrac{Q}{b} = \dfrac{22}{6} = 3.67 \ m^3/s/m'$

$\therefore E = y + \dfrac{(3.67)^2}{2*9.81*y^2} = y + \dfrac{1}{1.46y^2}$

$\Rightarrow \boxed{E = y + \dfrac{1}{1.46y^2}}$ $\quad y_c = \sqrt[3]{\dfrac{q^2}{g}} = \sqrt[3]{\dfrac{(3.67)^2}{9.81}} = 1.11$

y (m)	0.4	0.6	0.8	1	1.11	1.5	2	2.5	3
E (m)	4.68	2.5	1.87	1.68	1.66	1.8	2.17	2.61	3.07



From Graph :-

a) $y_c = 1.11$ m

b) $E_{min} = 1.66^m$

c) $y = 2^m \rightarrow E = 2.17^m$

d) $E = 2.5^m \rightarrow y_1 = 2.38^m$ & $y_2 = 0.6^m$

e) $y_1 = 1.5^m \rightarrow y_2 = 0.85^m$

f) $y = 0.6^m$ (super critical) & $y = 1.8^m$ (sub critical)

k) $S_{cr} = \dfrac{g n^2}{y_c^{1/3}} = \dfrac{9.81(0.017)^2}{(1.11)^{1/3}} = 0.0027$

g) at $y = 0.6^m$ & $Q = 22\ m^3/s$ & $n = 0.017$

$A = 6 * 0.6 = 3.6\ m^2$

$P = 6 + 2 * 0.6 = 7.2^m$

$Q = \frac{1}{n} R^{2/3} S^{1/2} A \Rightarrow 22 = \frac{1}{0.017}\left(\frac{3.6}{7.2}\right)^{2/3}(S)^{1/2} * 3.6$

$\Rightarrow S = 0.027 > \underset{\uparrow S_{cr}}{0.0027} \Rightarrow$ "Steep slope"

at $y = 1.8^m$

$A = 6 * 1.8 = 10.8\ m^2$

$P = 6 + 2 * 1.8 = 9.6$ m

$\Rightarrow 22 = \frac{1}{0.017}\left(\frac{10.8}{9.6}\right)^{\frac{2}{3}}(S)^{\frac{1}{2}}(10.8)$

$\Rightarrow S = 0.001 < \underset{\uparrow S_{cr}}{0.0027} \Rightarrow$ "Mild slope"

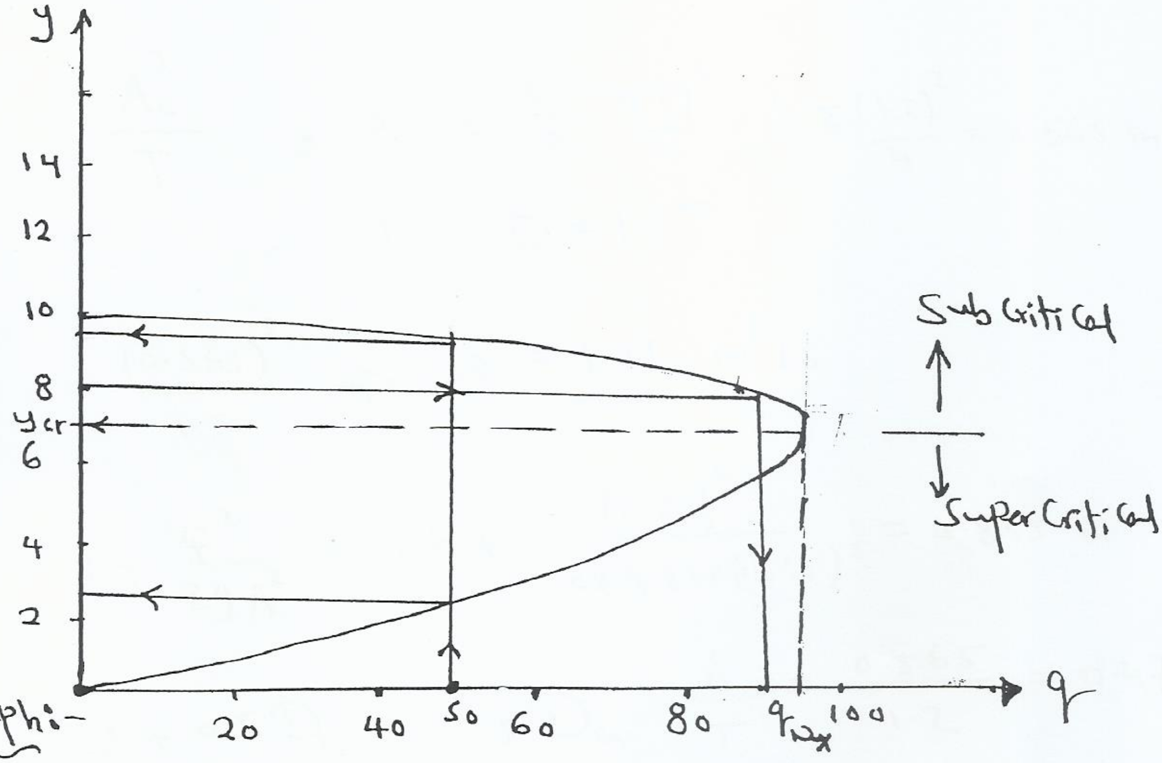
Question (9) :-

$E = 10$ ft & $b = 20$ ft

$q = y\sqrt{2g(E-y)}$

$\therefore q = y\sqrt{2*32.2(E-y)} = 8.02y\sqrt{(10-y)}$

y	0.5	1	2	3	4	5	6	7	8	9	10
q	12.36	24	45.4	63.3	78.6	89.7	96.24	97.3	90.8	72.2	0



From Graph:-

a) $y_c = 6.6$ ft & $q_{max} = 95\ ft^3/s/ft'$

b) at $y = 8$ ft $\Rightarrow q = 90.7 \Rightarrow Q = 20*90.7 = 1815.8\ ft^3/s$

c) $Q = 1000$ cfs $\Rightarrow q = \frac{1000}{20} = 50\ ft^3/s/ft'$

$y = 2.5$ ft $\rightarrow$ Super Critical

or

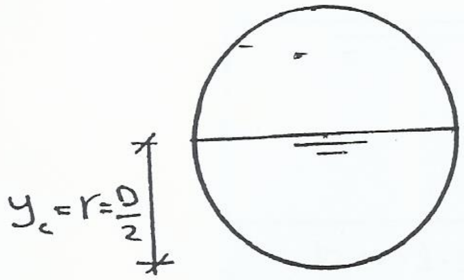
$y = 9.3$ ft $\rightarrow$ Sub Critical

Question (10) :-

$D = 1.2^{m}$

$y_c = \frac{D}{2}$ (Half full)

$= \frac{1.2}{2} = 0.6\ m$



Half Full

Req Q, E

$\frac{Q^2}{g} = \frac{A_c^3}{T}$, $A_c = \frac{1}{2}\ \frac{\pi D^2}{4} = \frac{1}{2}\pi\frac{(1.2)^2}{4} = 0.565\ m^2$

$T = D = 1.2^{m}$

$\frac{Q^2}{9.81} = \frac{(0.565)^3}{1.2} \Rightarrow Q = 1.21\ m^3/s$

$E_{min} = y + \frac{Q^2}{2gA^2} = 0.6 + \frac{(1.21)^2}{2*9.81*(0.565)^2} = 0.835\ m$

or $E_{min} = y + 0.5 D_{cr}$, $D_{cr} = \frac{A}{T} = \frac{0.565}{1.2} = 0.47^{m}$

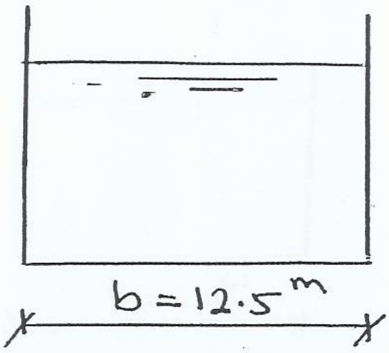
$\therefore E = 0.6 + 0.5*0.47 = 0.835^{m}$ ※

Question(11) :-

$Q = 32 \ m^3/s$

$y = 2^m$

$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(32/12.5)^2}{9.81}} = 0.87^m$



$\rightarrow y > y_c \rightarrow$ Flow is Subcritical

$n = 0.025$

Req S_{cr}

$A_c = 12.5 \ y_c = 10.87 \ m^2$

$P_c = 12.5 + 2y_c = 14.24^m$

$R_c = \frac{A_c}{P_c} = \frac{10.87}{14.24} = 0.76^m$

$Q = \frac{1}{n} R_c^{2/3} S_c^{1/2} A_c$

$32 = \frac{1}{0.025} (0.76)^{2/3} (S_{cr})^{1/2} * 10.87$

$\Rightarrow S_{cr} = 0.0078$

$*$ at $y = 2^m$, $Q = 32 \ m^3/s$

$S = ??$

$A = 12.5 * 2 = 25 \ m^2$

$P = 12.5 + 2*2 = 16.5 \ m$

$R = \frac{A}{P} = \frac{25}{16.5} = 1.515 \ m$

$Q = \frac{1}{n} R^{2/3} S^{1/2} A$

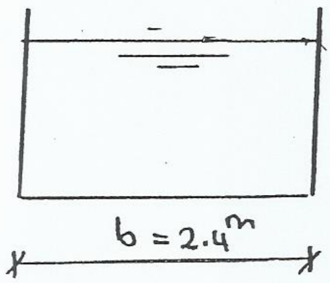
$32 = \frac{1}{0.025} (1.515)^{2/3} S^{1/2} * 25$

$\Rightarrow S = 0.00059 < S_{cr} \Rightarrow$ Sub Critical o.k

Question (12) :-

$E = 1.5 \ m$

$Q_{max} = ??$



$y_c = \frac{2}{3} E = \frac{2}{3} * 1.5 = 1 \ m$

$q_{max} = \sqrt{g y_c^3} = \sqrt{9.81 (1)^3} = 3.13 \ m^3/s/m$

$Q_{max} = q_{max} * b = 3.13 * 2.4 = 7.5 \ m^3/s$