(b) Derive an expression for escape velocity and calculate
on the Earth's surface

(Rawalpindi board 2001, 2002, Azad Kashmir 2002, Lahore 200

Q.9. Explain the phenomenon "inter conversion" of the kinetic and
energy.

(Rawalpindi board 1993, 2000, Azad Kashmir 2001, Sargodha 200

Q.10. Describe briefly the various sources of energy

(Gujranwala board 2001)

CHAPTER 5

CIRCULAR MOTION

INTRODUCTION:-

We have already studied velocity, acceleration, force and the laws of motion, all these are included in rectilinear motion (خطی حرکت). When a body moves in a circle, its motion is called circular motion (دائروی حرکت) or angular motion (زاویائی حرکت). The universe (کائنات) is full of a number of objects (اجسام) which move in nearly circular orbits (مدار). For example, the earth and planets (سیارے) revolve (گھومنا) around the sun, the moon moves around the Earth and motion of electrons around the nucleus (مرکزہ) is in nearly circular orbits.

When these objects move with uniform speed in circular paths, their direction is continually (لگاتار) changing. Since acceleration is a vector quantity. This change of direction means that their accelerations are not constant. The acceleration which makes the object to move in a circular orbit (path) is provided (مہیا کیا جاتا ہے) by some force. In case of planets, it is the gravitational force (تجاذبی قوت) which provides the necessary acceleration. For a car moving along a curve, it is provided by the force of friction (رگڑ کی قوت) between the road and the tyre. In case of circular motion of the electron around the nucleus is the Coulomb's force between the nucleus and the electron which is responsible for the acceleration. Such acceleration is called centripetal acceleration.

EXAMPLES:-

A stone whirled (گھمانا) around by a string, a car turning around a corner, satellites in orbits around the earth and motion of electrons around the nucleus are all examples of circular motion.

5.1. ANGULAR DISPLACEMENT (زاویائی ہٹاؤ)

DEFINITION:-

The angle through which a particle moves in a certain interval of time, while moving along a circle, is called its angular displacement, It is denoted by 'θ'.

EXPLANATION:-

Consider the motion of a single particle P of mass m in a circular path

of radius `r'. Let this motion be taking place by attaching the particle P at the end of a massless rigid rod of length `r' whose other end is pivoted (محور کرنا) at the centre `O' of the circular path as shown in fig 5.1 (a) As the particle is moving along the circular path,

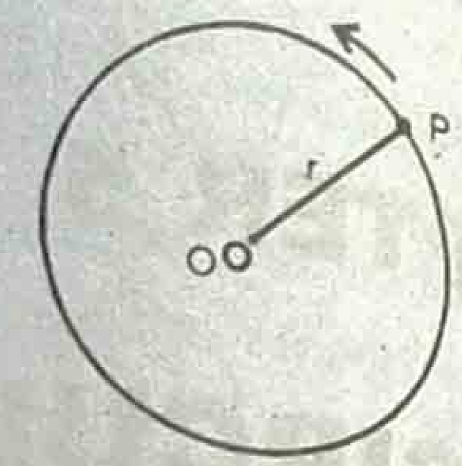


Fig. 5.1(a)

the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot (محور) O and is normal to the plane of rotation. Consider a system of axes as shown in fig 5.1 (b). The z-axis is taken along the axis of rotation with pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation.

While OP is rotating, let OP_1, be its position at any instant `t' making an angle `θ' with x-axis. After some time `t+Δt', let its position be $OP_2$ making angle `θ + Δθ' with x-axis as shown in fig 5.1 (c).

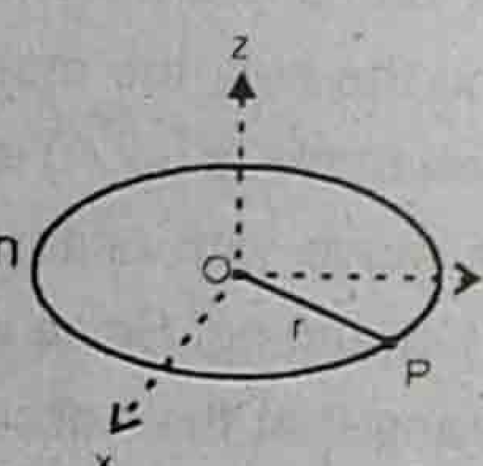


Fig. 5.1(b)

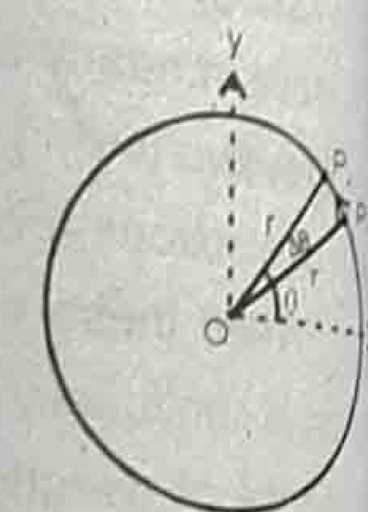


Fig. 5.1(c)

Thus,

Angle Δθ defines the angular displacement of OP during the time interval Δt.

If the rotation of OP is counter (anti) clockwise, the angular displacement Δθ is taken as positive. If the rotation is clockwise, Δθ is taken as negative.

For very small values of Δθ, the angular displacement is a vector quanty.

DIRECTION OF ANGULAR DISPLACEMENT:-

The direction of Dq is along the axis of rotation and is given by the right hand rule.

RIGHT HAND RULE:-

STATEMENT:-

Grasp the axis of rotation in right hand side with fingers curling (گھمانا) in the direction of rotation, the thumb points (اشارہ کرنا) in the direction of angular displacement, as shown in fig 5.1 d.

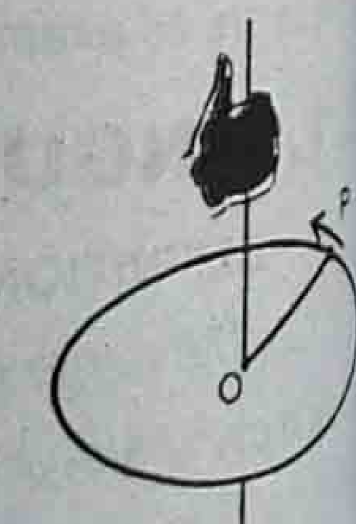


Fig. 5.1(d)

UNITS OF ANGULAR DISPLACEMENT:-

Angular displacement is measured in three units, namely

(i) Degrees
(ii) Revolution
(iii) Radian.

(i) **DEGREE:- (ڈگری درجہ)**

When a rotating object completes one revolution (چکر), it subtends (بنانا) an angle of 360 degrees at the centre of its circular path and thus its angular displacement is 360o. So, whole circle is divided into 360 equal divisions and each division is called one degree i.e.1o.

(ii) **REVOLUTION:-**

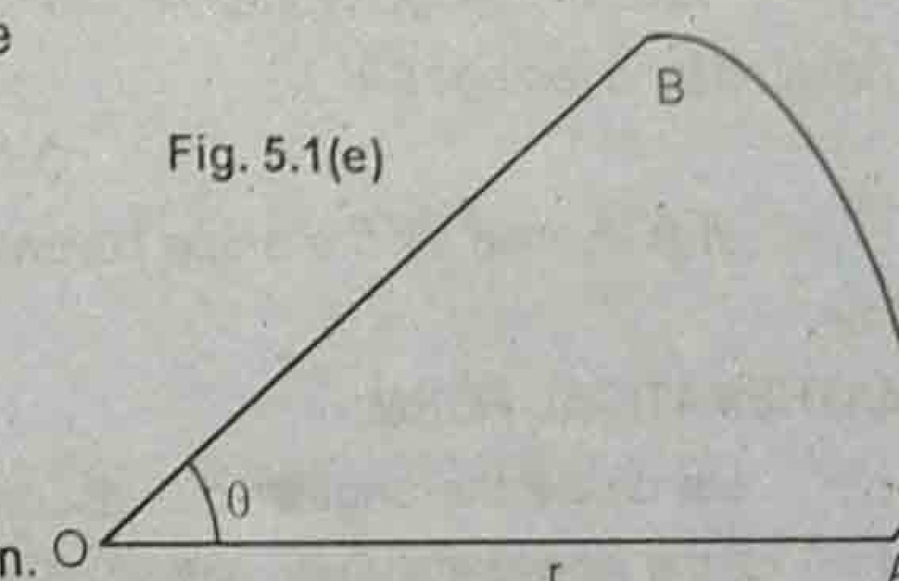
If a body completes one round trip along the circumference of a circle, then one round trip is said to be one revolution i.e. 1 revolution = 360^0 (or 2π)

RADIAN:-

It is the angle subtended at the centre of a circle by an arc (قوس) equal in length to its radius (نصف قطر) as in fig.fig 5.1 (c)

Arc `AB' is equal in length to the radius of circle and it makes an angle <$\hat{A}$OB at centre `q' which is one radian.

Fig. 5.1(e)



Advantage of Radian:-

If we know the angle in radians, we can easily find the length of an arc which subtends this angle at the centre.

RELATION BETWEEN ARC LENGTH, ANGLE SUBTENDED BY IT AND RADIUS (S = rθ)

Consider an arc of length `S' of a circle of radius r (as shown in fig 5.2) which subtends an angle `θ' at the centre of the circle given in radians (rad) as

or $$\theta = \frac{\text{arc length}}{\text{radius}} \text{ rad}$$

or θ = S/r rad

or S = rθ .... (1)

(where θ is in radians)

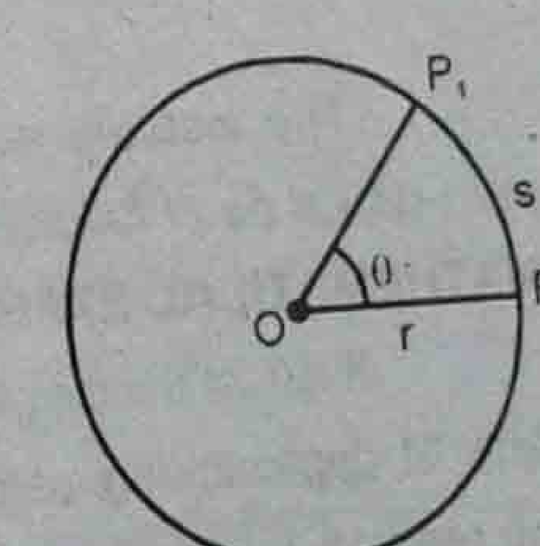


Fig. 5.2

RELATION BETWEEN RADIAN AND DEGREE:-

If OP is rotating in the above fig 5.2, the point P covers a distance S = 2πr in

one revolution of P in radian, it would be by using the relation

$$\theta = \frac{S}{r} = \frac{2\pi r}{r} = 2\pi \text{ radians}$$

So 1 revolution = 2π radian = 360^o

or 1 radian = $360^o/2\pi$

$$= \frac{180^o}{22/7} = 180 \times \frac{7}{22} = 57.3^o$$

Hence 1 radian = 57.3^o .... (2)

The equation (2) gives method of converting degrees into radians.

5.2. ANGULAR VELOCITY (زاویائی رفتار)

Sometimes we want to know how fast or how slow a body is rotating. It is found by its angular velocity.

DEFINITION:-

Rate of change of angular displacement is called angular velocity. It is denoted by ω (omega اومیگا).

OR

It is defined as the angle covered (طے کیا ہوا) by a rotating body in unit time.

MATHEMATICAL FORM:-

Let $\Delta\theta$ be the angular displacement during the time interval Δt (as in fig 5.1 c), the average angular velocity during this interval is given by

$\omega_{av} = \Delta\theta/\Delta t$ .... (1)

INSTANTANOUS ANGULAR VELOCITY:-

If the angular velocity is not uniform during the interval of time `Δt' then we can find the instantanous angular velocity at any instant.

DEFINITION:-

It is defined as the angular displacement in a very small interval of time.

OR

The instantaneous angular velocity `ω' is defined as the limit of the ratio $\theta\Delta$ /Δt as Δt following instant t, approaches (قریب پہنچنا) to zero.

MATHEMATICAL EXPRESSION:-

If $\Delta\theta$ is the angle described by the body during very short interval of time Δt approaching zero, then its instantaneous angular velocity is given by the relation

$$\omega_{ins} = \text{Lim}_{\Delta t \to o} \frac{\Delta\theta}{\Delta t} \quad \text{......................... (2)}$$

In the limit when Δt approaches zero, the angular displacement would be infinitesimally (لا محدود) small. The above equation shows that Δt is so small

that $\Delta\theta$ and Δt approach zero.

As angular velocity and instantaneous velocity are vector quantites. Their direction is along the axis of rotation and is given by the right hand rule as already described in the last article.

UNITS OF ANGULAR VELOCITY:-

(i) The SI unit of angular velocity is radian per second or written as rad s^{-1}.

Sometimes, other units of angular velocity are also expressed as.

(ii) Revolutions per second or written as rev s^{-1}

(iii) Degrees per second or written as deg s^{-1}

5.3 ANGULAR ACCELERATION (زاویائی اسراع)

When we switch on an electric fan in our room, we see that angular velocity of the fan goes on increasing slowly. In our general talk, we say that it has an angular acceleration

DEFINITION:-

Angular acceleration is defined as the rate of change of angular velocity. It is denoted by α (alpha. الفا).

MATHEMATICAL FORM:-(ریاضی کی شکل میں)

Let ω_i and ω_f be the values of instantaneous velocity of a rotating body at instant t_i and t_f, the average angular acceleration during the interval $(t_f - t_i)$ is given by

$$\text{Average angular acceleration} = \frac{\text{Change in angular velocity}}{\text{time interval}}$$

or

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t} \quad \text{......................... (1)}$$

or

$$\boxed{\alpha_{av} = \frac{\Delta\omega}{\Delta t}} \quad \text{......................... (2)}$$

INSTANTANEOUS ANGULAR ACCELERATION

DEFINITION:-

It is defined as the angular velocity `$\Delta\omega$' during a very small interval of time `Δt' approaching zero.

OR

Instantaneous angular acceleration is defined as the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches zero.

Mathematically, it is given by

$$\vec{a} = \text{Lim}_{\Delta t \to o} \frac{\Delta w}{\Delta t} \quad \text{......................} \quad (3)$$

Corollary:-(نتیجہ)

If angular acceleration is uniform, both average and instantanous accelerations are equal.

DIRECTION OF ANGULAR ACCELERATION:-

Angular acceleration is a vector quantity whose direction along axis of rotation is given by right hand rule as described earlier.

UNITS OF ANGULAR ACCELERATION:-

It is measured in rad. s^{-2} in SI units.

While other units are rev s^{-2} and degrees s^{-2} or degs^{-2}

ROTATION OF RIGID BODY:-

Consider the rotation of a rigid body as shown in the fig. 5.3. Imagine a point `P' on the rigid body. The line `OP' is the perpendicular drawn on the axis of rotation from the point P. It is usually referred as reference line. If the body totates, the line OP also rotates with it with the same angular velocity and angular acceleration. Thus the rotation of a rigid body can be described by the rotation of the reference line OP. In future while dealing with (گھماؤ) rotation of rigid body, we will replace it by its reference line OP.

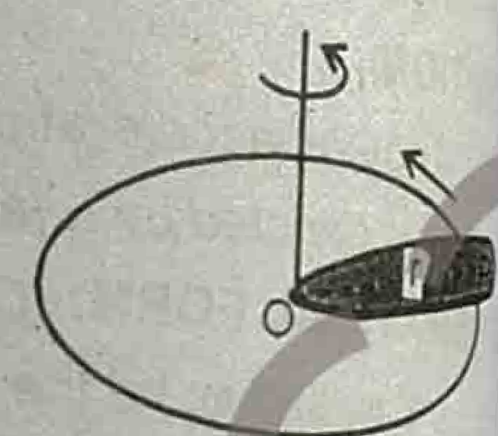


Fig. 5.3

5.4. RELATION BETWEEN ANGULAR AND LINEAR VELOCITIES (OR TO PROVE y = rω)

In order to (کی غرض سے) find the relation between linear and angular velocities we must consider a rigid body rotating about z-axis with an angular velocity as shown in fig 5.4 (a)

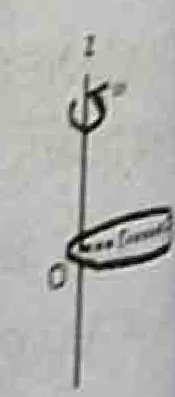


Fig. 5.4(a)

Consider a point P in the rigid body at a perpendicular r distance r from the axis of rotation. Here, OP indicates the reference line of the rigid body. When the rigid body rotates, thepoint `P' also moves in a circle of radius `r' with a linear velocity `v' while the reference line `OP' rotates with angular velocity `w' as shown in fig 5.4 (b). As we know that axis of rotation is fixed, then the direction of `w' always remains the same. Therefore, we shall deal with the magnitudes of angular velocity `w' and linearvelocity `v'.

Now during the circular motion, the point `P' moves

through a distance $P_1P_2 = \Delta S$ in a time interval Δt while the reference line `OP' has an angular displacement $\Delta\theta$ radian during this interval.

As we know the relation

$$S = r\theta$$

or

$$\Delta S = r\Delta\theta \quad \text{......................} \quad (1)$$

Dividing both sides of equation (1) by Δt, we get

$$\frac{\Delta S}{\Delta t} = \frac{r\,\Delta\theta}{\Delta t} \quad \text{......................} (2)$$

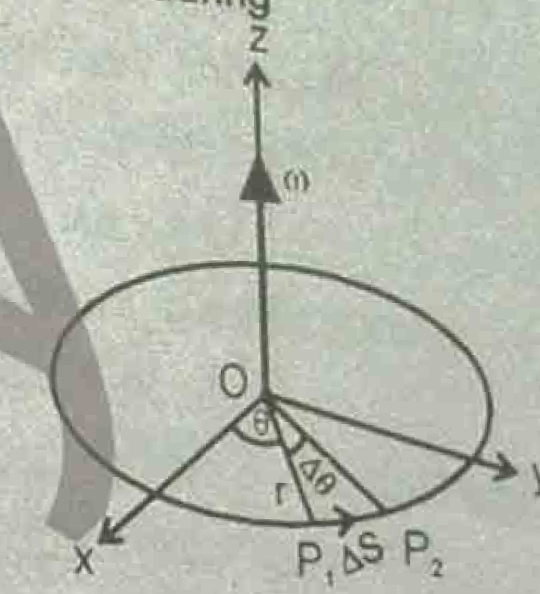


Fig. 5.4(b)

Taking its limits as $\Delta t \to o$, we get (Δt appraching to zero)

$$\text{Lim}_{\Delta t \to o} \frac{\Delta S}{\Delta t} = \text{Lim}_{\Delta t \to o} \frac{r\,\Delta\theta}{\Delta t} = r\text{Lim}_{\Delta t \to o} \frac{\Delta\theta}{\Delta t} \quad \text{..................} \quad (3)$$

using the definitions of instantancous linear and angular velocities, as the limit$\Delta t \to o$, the ratio `$\Delta S/\Delta t$' represents the instantaneous linear velocity `v' and the ratio $\Delta\theta/\Delta t$ represents the instantanous angular velocity `w'. Thus, we can write

$$\text{Lim}_{\Delta t \to o} \frac{\Delta S}{\Delta t} = v$$

and

$$\text{Lim}_{\Delta t \to o} \frac{\Delta\theta}{\Delta t} = \omega$$

putting these values in equation (3) we get

$$\boxed{v = r\omega} \quad \text{........................} \quad (4)$$

The equation (4) shows the relation between the magnitudes of linear and angular velocities.

In fig 5.4 (b), it can be seen that the point P is moving along the arc P_1P_2 in the limit when $\Delta t \to o$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus, the velocity with which point P is moving along the circle has magnitude `v' and its direction is always along the tangent to the circle. That is why the linear velocity of the point P is also known as tangent velocity.

5.4. (b) RELATION BETWEEN LINEAR AND ANGULAR ACCELERATIONS:-

The relation, v = rw shows that if the reference line OP is rotating (in fig 5.4 b) with an angular acceleration `a' then point P will have a linear or tangential acceleration.

As we know the relation

$$v = r\omega \quad \text{.................... (1)}$$

Considering the changes in linear and angular velocities, the equation (1) can be written as

$$\Delta v = r\,\Delta\omega \quad \text{...................... (2)}$$

Dividing both sides of the equation equ. (2) by Δt, we get

$$\Delta v/\Delta t = r\,\Delta\omega/\Delta t$$

Taking limit $\Delta t \to o$ on both sides so that Δt approaches to zero, we get

$$\text{Lim}_{\Delta t\to o} = \frac{\Delta v}{\Delta t} = \text{Lim}_{\Delta t\to o} \frac{r\,\Delta\omega}{\Delta t}$$

or $$\text{Lim}_{\Delta t\to o} \frac{\Delta v}{\Delta t} = r\,\text{Lim}_{\Delta t\to o} \frac{\Delta\omega}{\Delta t} \quad \text{.................... (3)}$$

But we know that

$$\text{Lim}_{\Delta t\to o} \frac{\Delta v}{\Delta t} = a_t \text{ (linear acceleration)}$$

and $\text{Lim}_{\Delta t\to o}\ \Delta\omega/\Delta t = \alpha$ (angular acceleration)

putting these values in equation (3) we get

$$\boxed{a_t = r\alpha} \quad \text{........................ (4)}$$

where α is the angular acceleration of point P.

The equation (4) shows the relation between the magnitudes of linear and angular accelerations.

IN VECTOR FORM:-

The equation (4) can be written in the vector form as

$$\vec{a} = \vec{\alpha} \times \vec{r}$$

NOTE:-

Both equations (1) and (4) show that the points which are at different displacements from the axis do not have the same speed or acceleration, but all points on a rigid body about a fixed axis have the same angular displacement. That is why we describe the motion of the entire body in a simple way.

5.4 (c) EQUATIONS OF ANGULAR MOTION:-

The equations of angular motion are exactly anaolgous ([illegible]) to those in linear motion except that θ, ω and α have replaced s, v and a respectively. The linear and angular accelerations together are given below:-

Linear Equations of Motion	Angular Equations of Motion	
$V_f = V_i + at$	$\omega_f = \omega_i + \alpha t$	(i)
$2aS = V_f^2 - V_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$	(ii)
$S = V_i t + 1/2\ at^2$	$\theta = \omega_i t + 1/2\ \alpha t^2$	(iii)

The angular equations (i), (ii), (iii) hold true only in the case when the axis of rotation is fixed, so that all the angular vectors have the same direction. Hence, they are used as scalars.

EXAMPLE 5.2:-

An electric fan rotating at 3 revs^{-1} is switched off. It comes to rest in 18.0s. Assuming deceleration ([illegible]) to be uniform find its value. How many revolutions did it turn before coming to rest?

SOLUTION:-

DATA:-

Initial angular velocity = ω_i = 3.0 revs^{-1}

final angular velocity = ω_f = 0

Time = t = 18.0 s

TO FIND:-

Angular acceleration = α = ?

Angular displacement = θ =?

FORMULA:-

(i) $$\alpha = \frac{\omega_f - \omega_i}{t}$$

(ii) $$\theta = \omega_i t + 1/2\ \alpha t^2$$

CALCULATIONS:-

using the formula of angular acceleration

(i) $$\alpha = \frac{\omega_f - \omega_i}{t}$$

Putting the values, we get

$$\alpha = \frac{0 - 3}{18} = -0.167 \text{ revs}^{-2}$$

Hence, $\boxed{\alpha = -0.167 \text{ revs}^{-2}}$ Ans.

we also know that

Angular displacement = $\theta = \omega_i t + 1/2\ \alpha t^2$

putting the values, we get

$$\theta = 3.0 \times 18 + 1/2\,(-0.167)\times(18)^2$$
$$= 54 + (-0.083)\times 324$$
$$= 54 - 26.4 = 27 \text{ rev}$$

or $\boxed{\theta = 27 \text{ rev.}}$ Ans.

RESULT:- No of revolutions of electric fan before to coming to rest is 27.

Q.1. You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

Ans:- We do not fall down even when we are upside down due to the centripetal force which is acting on us. This force keep us safe from falling down.

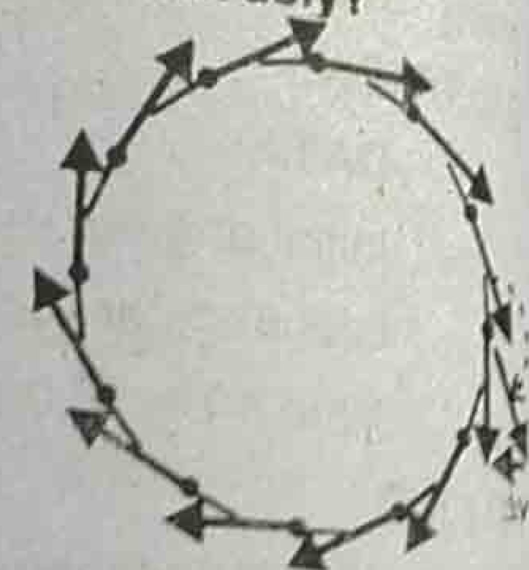
Q.2. Do you know; what happens as the wheel turns through an angle?

Ans:- As the wheel turns through an angle, it lays out a tangential distance

$$S = r.$$

Q.3. Dose the direction of circular motion change continuously?

Ans:- Yes, the direction of motion changes continuously in circular motion, as shown in fig.



5.5 CENTRIPETAL FORCE;-

DEFINITION:- *When a body moves in a circle with constant speed, the force which keeps the body in the circular path and always directed (سمت مرکز) towards the centre of the circle is called centripetal force.*

OR

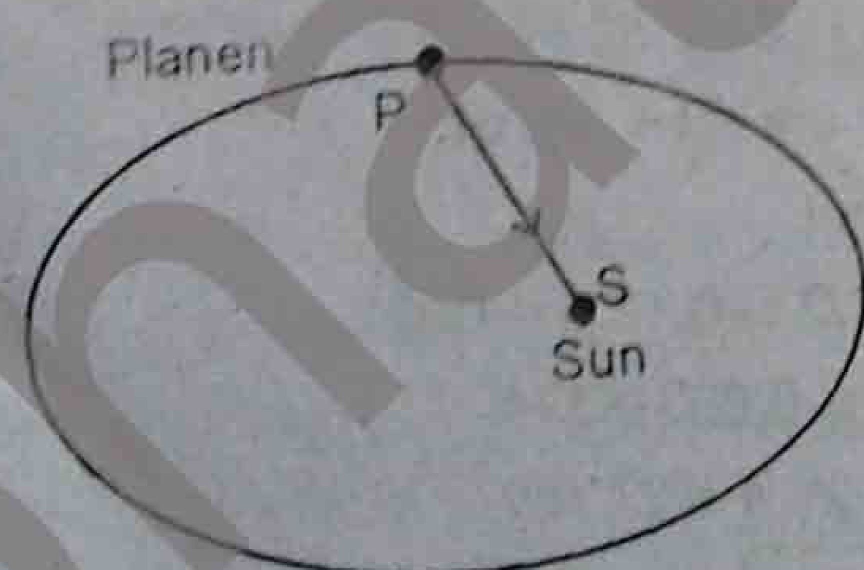
The force needed to bend the normally straight path of the particle into circular path is called the centripetal force.

EXPLANATION:-

According to Newton's first law of motion, a body can move along a straight line with uniform velocity only if no net force acts on it. For uniform circular motion, it must be under the continuous influence (اثر) of some force that changes the direction of velocity of the body at every instant (لمحہ) and thus produces the acceleration in the body. The centripetal force is always needed if the body is to be maintained (برقرار رکھنا) in its circular path. The presence of this force can by shown by the following examples.

Examples:-

(1) When a planet 'P' is moving around the sun 'S' in a nearly circular orbit (مدار), the centripetal force is provided by the gravitational attraction between 'S' and 'P' as shown in fig



(h)

(2) Motion of nuclear particles in accelerators (ذرات تیز کرنے والے), fiywheels spinnings on the the shafts (لمبا دھرا), motion of artifical and natural satellites are the example of centrictal force.

(3) A ball is tied at one end of the string while the other end is held in hand. Now this ball is whirled (گھمایا جاتا ہے) in horizontal circle. In order to (کی غرض سے) keep the ball along the circular path, the string must be pulled inward (اندر کی طرف) with a force which produces acceleration in the ball directed towards the hand. If the string breaks, when the ball is at the point A, in fig 5.5.(b) the ball will follow the straight line AB. It means that the ball flies off along the tangent to the circular path.

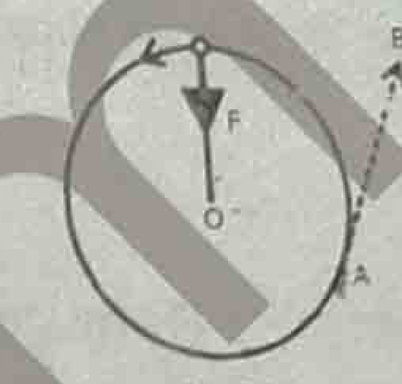


Fig. 5.5(b)

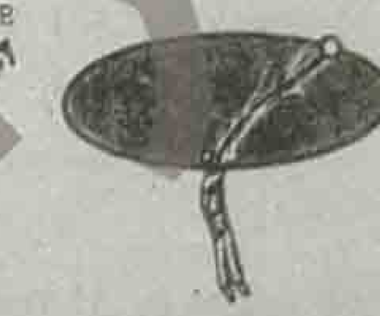


Fig. 5.5(a)

The fact is that unless a string pulls the ball towards the centre of the circle with a force, as shown in fig 5.5 (a), the ball will not continue along the circlular path.

CENTRIPETAL ACCELERATION (مرکز مائل اسراع)

DEFINITION:-

The acceleration produced by the centripetal force is always directed towards the centre of a circular path, therefore it is called centripetal acceleration.

OR

The instantaneous acceleration of an object travelling with uniform speed in a circle is directed towards the centre of the circle and is called centripetal acceleration.

EXPLANATION:-

If a body moves in a circle with constant speed, the magnitude of the velocity remains constant but its direction changes continuously. Due to this change in velocity, the body must have an acceleration which is directed towards the centre of the circle. Such an acceleration of the body directed towards the centre of the circle is known as centripetal acceleration.

DERIVATION OF EXPRESSION FOR CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE:-

Consider a particle moves from A to B with uniform v as shown in fig 5.6 (a), the velocity of the particle changes its direction but not its magnitude.

The change in velocity is shown in fig 5.6 (a)
Hence, the acceleration of the particle is

$a = \Delta v/\Delta t$ .... (1)

where Δt is the time taken by the particle to move from A to B. Let the velocity at A is v_1 and at B the velocity is v_2. Since the speed of the particle is v_1 so the time taken to cover a distance `S' (AB) as shown in fing 5.6 (a) is

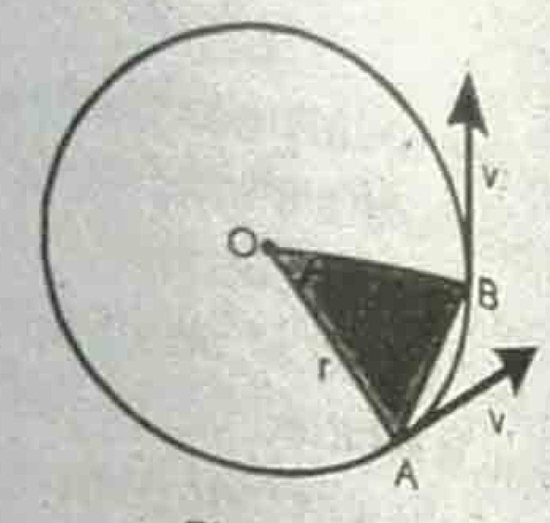


Fig. 5.6(a)

$$\Delta t = \frac{S}{v} \qquad (S = vt)$$

Putting the value of Δt in equation (1) we get

$$a = \frac{\Delta v}{S/v} = v\frac{\Delta v}{S}$$

or $$a = \frac{v\,\Delta v}{S} \quad \text{.................................} \ (2)$$

Let us now draw a triangle PQR such that PQ is parallel and equal to v_1 and PR is parallel and equal to v_2 as shown in fig 5.6 (b). As we know that the radius of the circle is perpendicular to its tangent, so OA (radius of circle) is perpendicular to v_1 and OB is perpendicular to v_2 as in fig 5.6 (b) Therefore, angle AOB equals the angle QPR between v1 and v_2.
Also we know that in magnitude

$v_1 = v_2 = v$

and OA = OB (both are radii)

As <AOB = <QPR

Thus, both the triangles are isosceles (مساوی الساقین) so their remaining angles <A,<B and <Q, <R should also be equal.

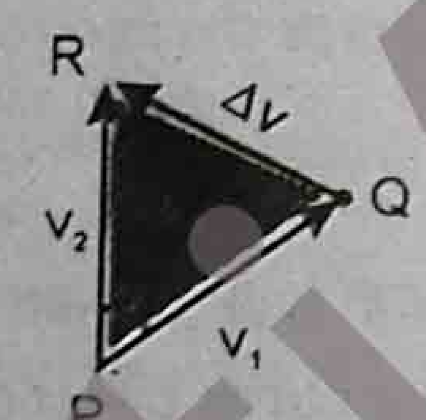


Fig. 5.6(b)

Therefore, the triangles OAB and PQR are similar. Hence, we can write

$$\frac{QR}{PR} = \frac{AB}{OB} \qquad (PR = v_1 = v)$$

or $$\frac{\Delta v}{v} = \frac{AB}{r} \quad \text{............................} \ (3)$$

If the point B is close to the point A on the circle, as will be the case when $\Delta t \to 0$, the arc AB is of nearly the same length as the line AB. So we can take AB = S.

Now putting the value of AB in equation (3) we get

$$\frac{\Delta v}{v} = \frac{S}{r}$$

or $$\Delta v = \frac{Sv}{r} \quad \text{........................} \ (3)$$

Now putting the value of Δv in equation (2) we have

$$a = v\,\frac{Sv}{r}\cdot\frac{1}{S} = \frac{v^2}{r}$$

Hence $$a = v^2/r$$

where a is the instantaneous acceleration. As this acceleration is produced by the centripetal force, it is called centripetal acceleration denoted by a_c. Therefore, from the equation (4) it can be written as

$$\boxed{a_c = \frac{v^2}{r}} \quad \text{.........................} \ (5)$$

The equation (5) gives the magnitude of the centripetal acceleration.

DIRECTION OF CENTRIPETAL ACCELERATION:-

Since PQ is perpendicular to OA and PR is perpendicular to OB, So QR (third side of the triangle PQR) must be perpendicular to AB which is third side of the second triangle OAB. Thus QR is parallel to the perpendicular bisector (نصف کرنیوالا) of AB. As the acceleration of the object moving in the circle is parallel to Δv when $AB \to o$, so centripetal acceleration is directed along radius towards the centre of the circle.

MAGNITUDE OF CENTRIPETAL FORCE:-

The magnitude of the centripetal force acting on the particle of mass `m' in uniform circular motion is obtained as follows:-

According to Newton's second law of motion,

$$F_c = ma_c \quad \text{.........................} \ (6)$$

But $a_c = v^2/r$

Puting this value of `a_c' in equation (6), we get

$$\boxed{F_c = mv^2/r} \quad \text{......................} \ (7)$$

The equation (7) gives the magnitude of centripetal force.

In angular measure. this equation can be written as

$$F_c = \frac{mr^2\omega^2}{r} \qquad (v = r\omega)$$

or $$\boxed{F_c = mr\omega^2} \quad \text{..............................} \ (8)$$

Q.1. Why are the banked tracks needed for turns?

Ans:- Banked tracks are needed for turns that are taken so quickly that friction alone cannot provide energy for centripetal force.

NOTE:- FOR YOUR KNOWLEDGE:-

Curved flights at high speed requires a large centripetal force that makes the stunt (کرتب) dangerous even if the air planes are not so close.

EXAMPLE 5.3

A 1000 kg car is turning round a corner at 10 ms^{-1} as it travels along an arc of a circle. If the radius of the circular path is 10m, how large a force must be exerted (ٹھ) by the pavement (سڑک راستہ) on the tyres to hold the car in the circular path?

SOLUTION:-

DATA:-

Mass of car = m = 1000kg

Velocity of car = v = 10 ms^{-1}

Radius of circular path = r = 10 m

TO FIND:-

centripetal force = ?

FORMULA:-

$$F_c = \frac{mv^2}{r}$$

CALCULATIONS:-

Using the formula for centripetal force

$$F_c = \frac{mv^2}{r}$$

Putting the values, we get

$$F_c = \frac{1000 \times (10)^2}{10} = \frac{1000 \times 10 \times 10}{10}$$

$= 1.0 \times 10^4$ $kgms^{-2}$ (1 kg ms^{-2} = 1N)

or $\boxed{F_c = 1.0 \times 104N}$ Ans.

RESULT:- The force exerted by the pavement is 1.0×10^4 N.

EXAMPLE 5.4:-

A ball tied to the end of a string is swung in a vertical circle of radius `r' under the action of gravity as shown in fig 5.7. What will be the tension in the string when the ball is at the point A of the path and its speed is `v' at this point? ***(Lahore 1988, Sargodha 1985)***

SOLUTION:-

DATA:-

As the ball moves in a circle, so the force acting on the ball must provide the required centripetal force. At point A, two forces are acting on the ball.

(i) Tension `T' in the string

(ii) weight `w' of the ball

These forces act along the radius at point A, so their vector sum must furnish the required centripetal force.

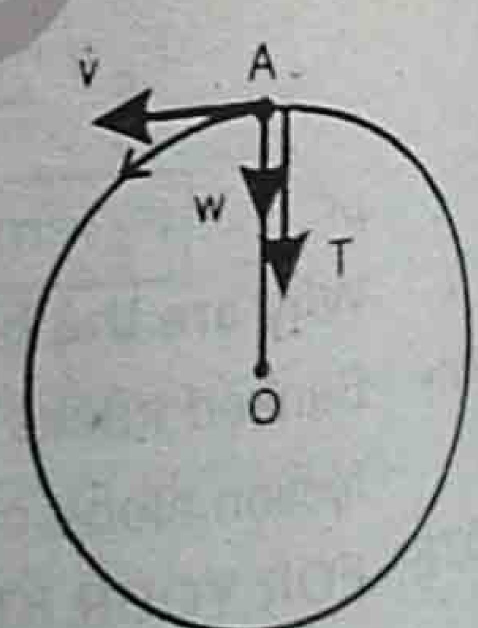


Fig. 5.7

TO FIND:-

Tension in the string = ?

FORMULA:-

$$T + w = \frac{mv^2}{r}$$

CALCULATIONS:-

using the formula

$$T + w = \frac{mv2}{r}$$ (T and w in the same direction)

or $$T = \frac{mv2}{r} - w$$

But w = mg

$$T = \frac{mv^2}{r} - mg$$

or $T = m(v^2/r - g)$

If $v^2/r = g$, then tension `T' will be zero and the centripetal force is equal to the weight of the ball.

RESULT:-

The tension in the string is zero and the centripetal force is just equal to the weight.

5.6. MOMENT OF INERTIA

DEFINITION:-

The moment of inertia of a body about an axis is equal to the sum of the products of the masses of particles in the body and the squares of their respective distances from the axis of rotation.

OR

It is defined as the product of mass of particle and square of its perpendicular distance from axis of rotation. It is denoted by I.

Mathematically it is written as $I = mr^2$

where m is the mass of the particle and r is the perpendicular distance of particle from the axis of rotation.

EXPLANATION:-

Consider a mass attached to a light (ہلکی) rod, it can rotate about the pivot point (نقطہ محور) `O' which is frictionles as shown in fig 5.8. The mass of the rod is negligible (چھوڑ دینا). Let this system is in horizontal plane.

A force F is acting on the mass perpendicular to the rod and hence, this will accelerate (رفتار میں اضافہ) the mass according to the law of motion.

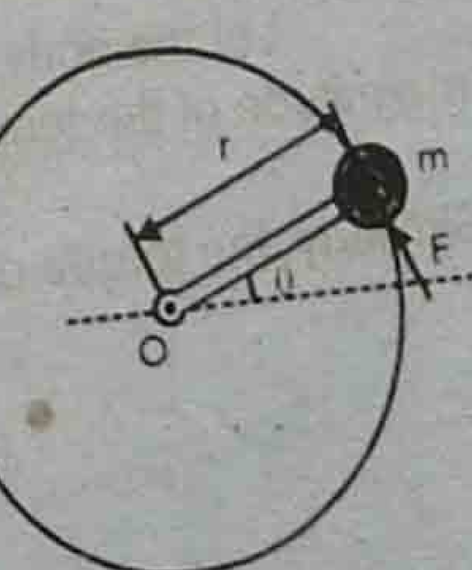


Fig. 5.8

F = ma .... (1)

This force will rotate the mass about `O'. Since tangential accelera[tion] is related to angular acceleration `a' by the equation

$a_T = r\alpha$ .... (2)

Putting the value of a_T in equation (1) we get

$F = mr\alpha$ .... (3)

Multiplying both sides of the equation (3) by r , we get

$rF = \tau = \text{torque} = mr^2 a$

$\therefore \quad \tau = mr^2\alpha \doteq I\alpha$

which is analogues (مشابہ) of the Newton's second law of motion.

$F = ma$

Here F is replaced by τ, a by a and `m' by mr^2. The quantity mr^2 is known as moment of inertia and is represented by *I*.

The moment of inertia plays the same role in angular motion as the ma[ss] in linear motion.

DEPENDENCE (انحصار)

Moment of inertia depends upon two things.

(1) mass `m'

(2) r^2 ,square of the perpendicular distance from the axis of rotation.

MOMENT OF INERTIA OF A RIGID BODY:-

In most of the rigid bodies, the mass distribution is not uniform. The rigi[d] body is made up of `n' small pieces of masses $m_1, m_2$ ................ mn at distanc[es] $r_1, r_2$ .............. $r_n$ frome the axis of rotation `O' as shown in fig 5.9.

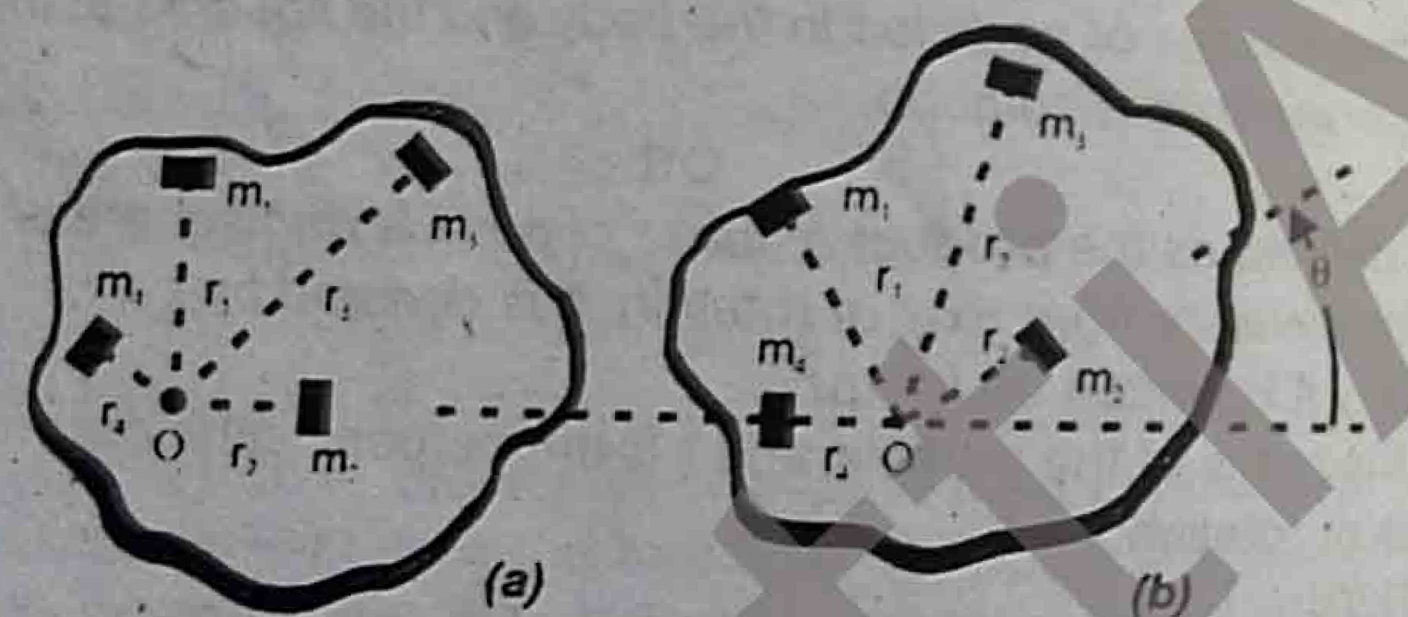


Fig. 5.9

Let the body be rotating with the angular acceleration `a' so the magnitude of the torque acting on m_1 given by

$\tau^1 = m^1 r_1^2 \alpha_1$

Similarly, the torque on m_2 is

$\tau^2 = m^2 r_2^2 \alpha^2$

....

....

$\tau_n = m_n r^2_n \alpha_n$

Since the body is rigid, so all the masses are rotating with the same angular

[ac]celeration α. Thus,

[to]tal torque `τ' is then given by torque

$\tau_{total} = (m_1r_1^2 + m_2r_2^2 + \ldots\ldots\ldots\ldots\ldots\ldots m_n r_n^2)\alpha$

[Wri]ting in the summation (Σ) form, we get

$\tau_{total} = (^n\Sigma^{i=1} m_i r_i^2)\ \alpha$

$\tau = I\alpha$ .... (1)

[Wh]ere I is the moment of inertia of the body and is written as

$I = {}^n\Sigma_{i=1} m_i r i^2$ .... (2)

[U]NIT:-

The SI unit of moment of inertia is kg m^2.

[D]IMENSIONS:-

As we know $I = Mr^2$

Dimensionally, it can be written as

$[I] = [M][L^2]$

or $[I] = [ML^2]$

[H]ence, dimensions of moment of inertia is $[ML^2]$

[5].7. ANGULAR MOMENTUM

DEFINITION:-

A particle is said to possess an angular momentum about a reference axis [if i]t so moves that its angular position changes relative to that reference axis.

OR

[Th]e angular momentum of an object is defined as the cross product of position [v]ector $\vec{r}$ with respect to (کے لحاظ سے) the axis of rotation and linear momentum $\vec{p}$ of [a]n object.

It is denoted by $\vec{L}$.

[R]ELATION BETWEEN ANGULAR MOMENTUM AND MOMENT OF INERIA

Consider a particle of mass m moving with velocity $\vec{v}$ having linear [m]omentum $\vec{p}$ as shown in fig 5.10 The angular momentum L of the particle [r]elative to the origin `O' is mathematically (ریاضی کی رو سے) given by

$\vec{L} = \vec{r} \times \vec{p}$ .... (1)

[W]here $\vec{r}$ is the position vector of the particle at [th]at instant relative to the origin `O'. Angular [m]omentum is a vector quantity. Its magnitude is

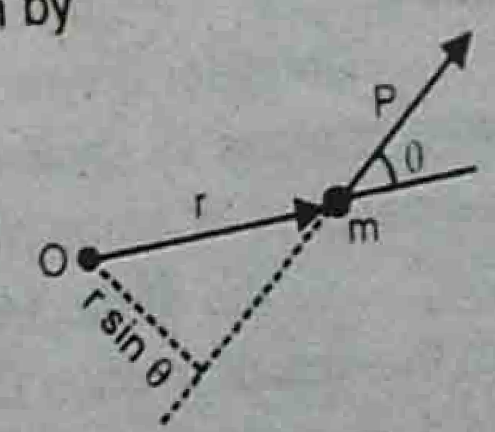


Fig. 5.10

$L = rp\sin\theta$

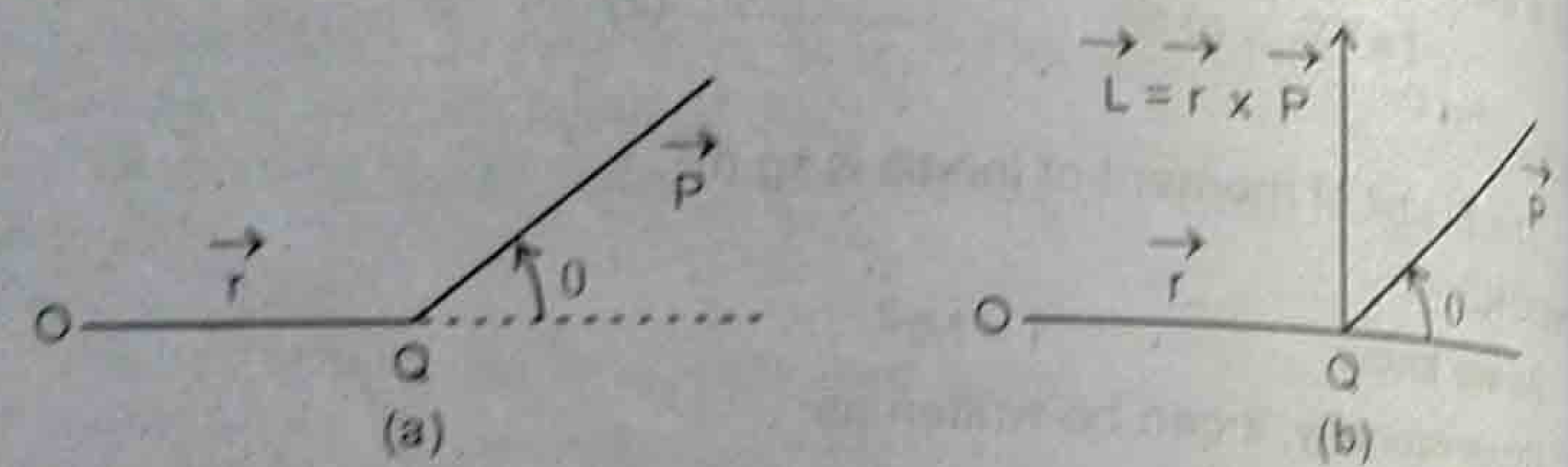
$= rm\,v\sin\theta \qquad (p = mv)$

or $\qquad L = m\,\vec{r}\vec{v}\sin\theta$ .... (2)

where q is the angle between r and p.

DIRECTION:-

As the angular momentum is vector quantity, so the direction of angular momentum $\vec{L}$ is perpendicular to the plane formed by $\vec{r}$ and $\vec{p}$ as shown in fig below.



Its direction is determined by the right hand rule of vector product as described earlier. If an object is rotating along certain axis, then direction of $\vec{L}$ will be along axis of rotation.

Case I;-Angular Momentum of a Particle:-

If the particle is moving in a circle of radius 'r' with uniform angular velocity 'w' then angle between r and tangential velocity is 90o. Hence magnitude of angular momentum from equation (2) becomes

$$L = m\,r\,v\sin 90 \qquad (\sin 90^0 = 1)$$

$$= mrv \text{ .... (3)}$$

But $\qquad v = r\omega$

Hnece, equation (3) can be written as

$$L = mr(r\omega) = mr^2\omega$$

$$\therefore L = mr^2\omega$$

where quantity mr2 is the moment of inertia and represented by I. Therefore

$$\boxed{L = I\omega} \text{ .... (4)}$$

Case II:- Angular Momentum of a Rigid Body:-

Consider a symmetric rigid body rotating about a fixed axis through the centre of mass as shown in fig 5.11. As the rigid body is made up of 'n' small pieces of masses m_1, m_2 .... mn at distances r_1, r_2, ... r_n from the axis of rotation O. Each particle of the rigid body rotates about the same axis in a circle with angular velocity 'ω'.

Fig 5.11

The magnitude of angular momentum due to mass $m_1 = L_1 = m_1r_1v_1$

The magnitude of angular momentum due to mass, $m_2 = L_2 = m_2r_2v_2$

....

....

The magnitude of angular momentum due to mass, mn

$$= L_n = m_nr_nv_n$$

As the direction of Ln is the same as that of ω. Therefore,

$$v_n = rn\omega$$

The angular momentum of the nth particle becomes

$$L_n = m_nr_n^2\omega$$

Total angular momentum of the rigid body is given as

$$L_{total} = L_1 + L_2 + \text{ .... } L_n$$

$$= m_1r_1v_1 + m_2r^2v_2 + \text{ .... } + m_nr_nv_n \text{ .... (6)}$$

But $\qquad v = r\omega$

Thus, equation (6) can be written as

$$L_{total} = m_1r^2{}_1\omega + m_2r_2^2\omega + \text{ .... } m_nr_n^2\omega$$

Writing in summation (â) form,we get

$$L_{total} = (^n\Sigma_{i=1} m_ir_i^2)\omega = I\omega$$

Hence $\qquad L = I\omega$ .... (7)

where I is the moment of inertia of the rigid body about the axis of rotation.

DIFFERENCE BETWEEN SPIN ANGULAR MOMENTUM AND ORBITAL ANGULAR MOMENTUM:-

Physicists usually (عموماً) describe a distinction between spin angular momentum (L_s) and orbital angular momentum (L_o).

The spin angular momentum is the angular momentum of a spinning (لٹو کی طرح گھومتا) body, while orbital angular momentum is associated (منسلک ہونا) with the motion of a body along a circular path.

The difference is shown in fig 5.12. In the usual circumstances concerning orbital angular momentum, the orbital radius is large as compared to the size of the body. Therefore, the body is considered to be a point size.

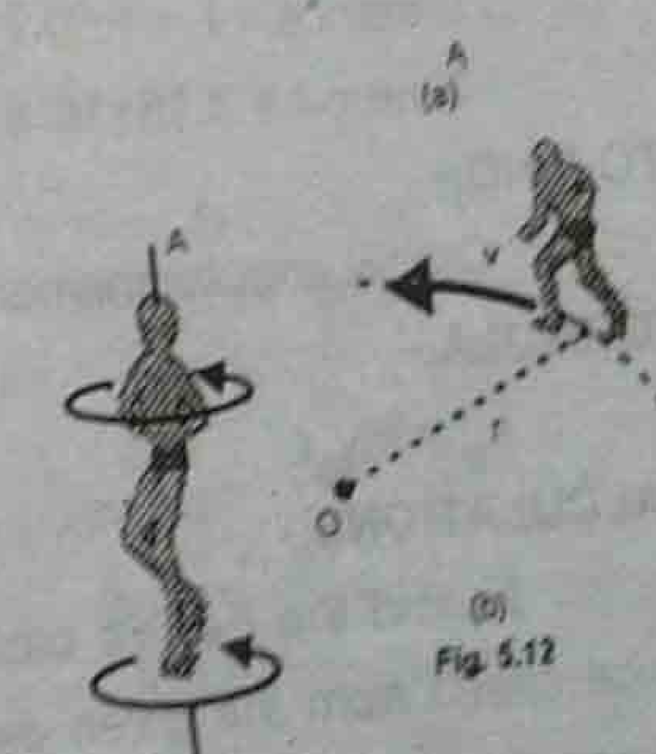


Fig 5.12

UNIT OF ANGULAR MOMENTUM:

The SI Unit of angular momentum is kg m^2s^{-1} or Js.

Derivation:- It can be found by

$$L = mvr = kg\,\frac{\times m \times m}{s}$$

$\therefore$ Unit of $L = kg\, m^2/s = kgm^2s^{-1}$

Alternative unit:-

$F = ma = kg \times m/s^2 = 1\ N$ (i.e. newton)

$\therefore$ $kg \times m/s^2 \times m = (1N) \times m$ (Multiply by m)

or $kg\ \frac{m^2}{s} = 1N \times m \times s = 1\ N \times ms$

But $1\ N\text{--}m = \text{Joule}$ $(F \times d = \text{work})$

$\therefore$ $Kgm^2s^{-1} = \boxed{Js}$

so, its alternative unit is $\boxed{Js}$

DIMENSIONS:-

As we know the unit of angular momentum is

$L = kgm^2s^{-1} = \text{Mass} \times (\text{distance})^2 \times (\text{Time})^{-1}$

Dimensionally, it can be wrtten

$[L] = [M][L^2][T^{-1}]$

or $[L] = [ML^2T^{-1}]$

EXAMPLE:- 5.5.

The mass of Earth is 6.00×10^{24} kg. The distance `r' from Earth to the Sun is 1.50×10^{11}m. As seen from the direction of the North Star, the Earth revolves counter -clockwise around the Sun. Determine the orbital angular momentum of the Earth about the sun, assumming that it traverses a circular orbit about the Sun once a year (3.16×10^7 s).

SOLUTION:-

DATA:-

Mass of Earth = $m = 6.00 \times 10^{24}$ kg

Distance = $r = 1.50 \times 10^{11}$ m

Time = $t = 3.16 \times 10^7 s$

TO FIND:-

Orbital angular momentum of Earth = L_o = ?

FORMULA:-

$L_o = mv_or$

CALCULATIONS:-

To find the Earth's orbital angular momentum, we must know first its orbital speed from the given data. When the Earth moves around a circle of radius r, it travels a distance of $2\pi r$ in one year, it orbital speed is given by

$$v_o = \frac{2\pi r}{t} \quad \text{.................... (1)}$$

orbital angular speed of Earth = $L_o = mv_or$ .... (2)

Putting the value of v_o from equ (1) into equation (2) we have

$$L_o = \frac{2\pi r^2 m}{t} \quad \text{...................... (3)}$$

Now putting the values in equations (3) we get

$$L_o = \frac{2 \times 3.14(1.50 \times 1011)2 \times (6.00 \times 1024)}{3.16 \times 107}$$

$$= \frac{2 \times 3.14 \times 1.50 \times 1.50 \times 1022 \times 6 \times 1024}{3.16 \times 107}$$

$$= \frac{2 \times 3.14 \times 1.50 \times 1.50 \times 6 \times 1046}{3.16 \times 107}$$

$$Lo = 2.67 \times 1040\ kgm^2s^{-1}$$

he sign is positive because the revolution is counter clock-wise.

.8. LAW OF CONSERVATION OF ANGULAR MOMENTUM

As in the case of linear motion, linear momentum remains constant if no xternal force acts upon. Similarly in rotatory motion, angular momentum remains onstant if no external torque acts upon it.

TATEMENT:-

It states that if no external torque acts on a system, the total angular omentum of the system remains constant.

Mathematically, it is expressed as

$L_{Total} = L_1 + L_2 + L_3 + \text{..........................} = \text{constant}$

The law of conservation of angular momentum is one of the fundamental inciples of Physics.

XPLANATION:-

The effect of the law of conservation of angular momentum is apparant) if a single isolated (الگ شدہ) spinning body changes its moment of inertia. his can be explained by taking the example of a diver (غوطہ زن) in fig 5.13.

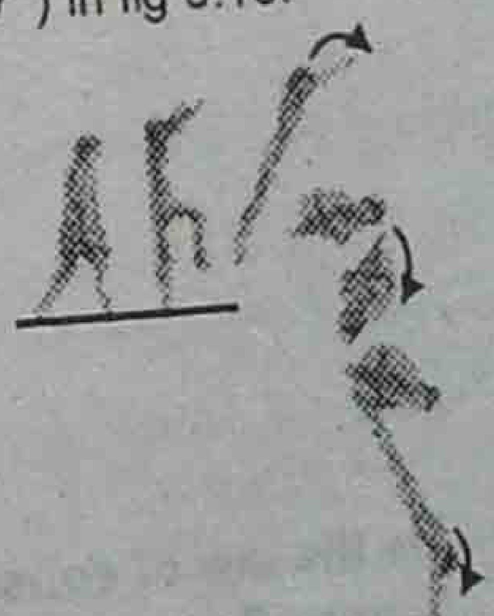


Fig. 5.13

The diver pushes off (دھکیلنا) the board with small angular velocity about a horizontal axis through e centre of gravity `G'. Upon jumping off the board, e divers legs and arms are fully extended (پھیلانا) hich means that diver has a large moment of inertia about this axis. The moment of inertia is considerably ) reduced to a new value `I_2' when the legs and ms into the closed tuck (سکیڑنا ۔ اندر ڈالنا) position.

As we know

$L = mr^2\omega$

In this case, the value of r is reduced thus the value of mr2 (moment of inerti[a]) decreases. Hence, the value of w2 must increase to keep the angular momen[tum] constant.

As the angular momentum is conserved, so

$$I_1\omega_1 = I_2\omega_2$$

Hence, the diver must spin (گھومنا) faster when moment of inertia becom[es] smaller to conserve angular momentum. In this way, the diver can make more so[m]mersaults (قلابازیاں) before entering the water.

DIRECTION OF ANGULAR MOMENTUM

The angular momentum is a vector quantity with direction along the axi[s] of rotation. The direction of angular momentum along axis of rotation also rema[ins] constant. This is illustrated (مثال دینا) by the fact given below

The axis of rotation of an object will not change its orientation (سمت) unless an external torque causes it to do so.

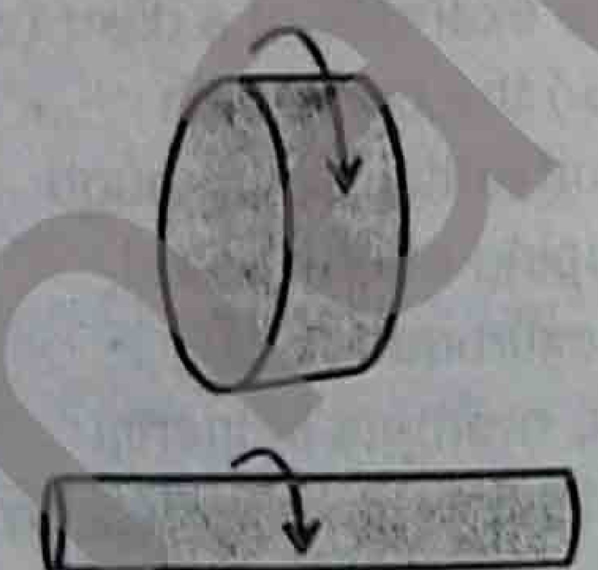
This fact is of great importance for Earth as it moves around the Sun. N[o] other sizeable (big) torque is experienced by the Earth, because the major (بڑی) force acting on it is the pull of the Sun. The earth;s axis of rotation, therefore, remains fixed in one direction with reference to the universe around us.

APPLICATIONS OF LAW OF CONSERVATION OF ANGULAR MOMENTUM

The law of conservation of angular momentum has many applications from creation of stars down to subatomic particles such as electrons. protons an[d] neutrons. Divers (غوطہ زن), ice skaters (برف پر چلنے والے), ballet dancers (ناچ گانا کرنے والے), and acrobates (بازیگر) and others make use of to show spectacular feat (تماشائی کرتب).

Q.1. Do you know? which cylinder has a greater rotational inertia?

Ans:- Two cylinder of equal mass. The one with the larger diameter has the greater rotational inertia.



Q.2. Is the law of conservation of angular momentum applicable to sports?

Ans:- Yes, the law of conservation of angular momentum is important in many sports, particularly in diving, gymnastics and ice- skating.

5.9.ROTATIONAL KINETIC ENERGY (گردشی حرکی توانائی)

DEFINITION:-

The energy due to the spinning (محوری گردش) of a body about an axis is called rotational kinetic energy.

(a) EXPRESSION (OR FORMULA) FOR ROTATIONAL KINETIC ENERGY:-

Consider a body spinning about an axis with constant angular velocity `ω' each point of the body is moving in a cicular path and, therefore, has some K.E.

In order to (کی غرض سے) determine the tatal K.E. of a spinning body (گھومتا ہوا) we suppose that the body is made up of a large number of tiny (بہت چھوٹے) pieces of masses m_1, m_2, m_3 .... which are situated (واقع ہونا) at distances r_1, r_2, r_3, ...respectively from the axis of rotation. As the body rotates with constant angular velocity `ω' so the angular speeds of all the particles will be the same `ω' but their linear speed `v' will be different. If a piece of mass mi is at a distance ri from the axis of rotation as shown in fig 5.14 the piece is moving in a circle with speed

$$v_i = r_i\omega$$

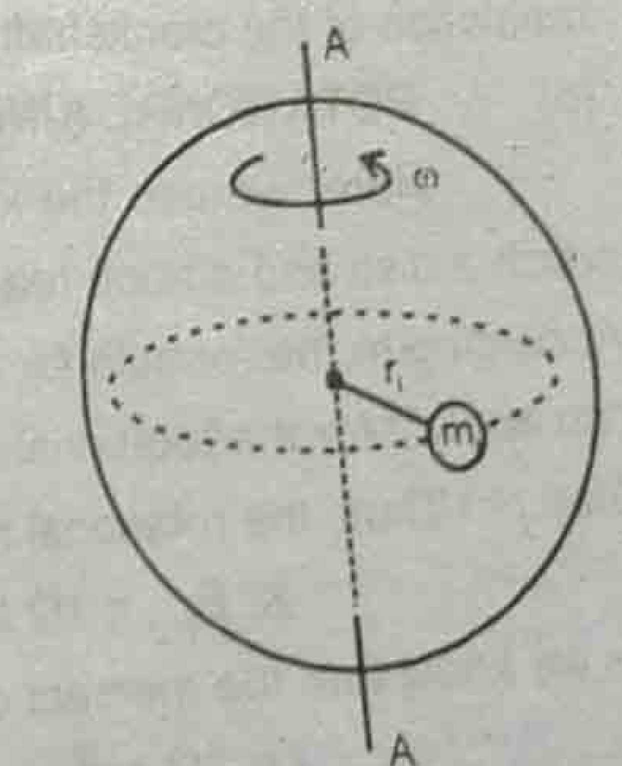


Fig. 5.14

Thus the kinetic energy of the piece is

$$(K.E)_i = \frac{1}{2} m_i v_i^2$$

$$= \frac{1}{2} m_i (r_i\omega)^2 \qquad (v = r\omega)$$

$$= \frac{1}{2} m_i r_i^2 \omega^2$$

Similarly, the kinetic energies of other pieces m_1, m_2, m_3 .... are

$$\frac{1}{2} m_1 r_1^2 \omega^2, \frac{1}{2} m_2 r_2^2 \omega^2, \frac{1}{2} m_3 r_3^2 \omega^2 \text{ ....}$$

∴ The rotational K. E. of the whole body is the sum of the kinetic energy of all the pieces. So we have

$$K.E._{rot} = \frac{1}{2}(m_1 r_1^2\omega^2 + m_2 r_2^2\omega^2 + m_3 r_3^2\omega^2 + \text{..................})$$

$$\frac{1}{2}\omega^2 (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \text{..................})$$

where $I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \text{..................})$

Hence, rotational kinetic energy is given by

$$K.E_{rot} = \frac{1}{2} I\omega^2 \quad \text{.......................} (1)$$

This expression for the kinetic energy of rotation is analogous (مشابہ) to the expression for the K.E. of linear motion i.e. $1/2\ mv^2$. Linear speed v has been replaced by the angular speed ω and 'm' has been replaced by moment of inertia 'I'.

PRACTICAL USE OF ROTATIONAL K. E.

Rotational kinetic energy is put to practical use by fly wheels, which are essential (لازمی) parts of many engines. A fly wheel stores energy between the power strokes of the pistons, so that the energy is distributed over the full revolution of the crankshaft (مشین کا دھرا) and hence, the rotation remains smooth.

(b) ROTATIONAL KINETIC ENERGY OF A DISC AND A HOOP.

Here, we use the idea of rotational K.E. to compare the velocity with which a disc and a hoop reach the bottom of an inclined plane (ڈھلوان سطح). According to the formula for the rotational K.E. of a body i.e $K.E._T = 1/2\ I\omega^2$, we can apply it for the rotational K. E. of disc.

Disc.:- Thus, the rotational K. E. of disc is

$$K.E_{rot} = 1/2\ I\omega^2$$

As we know that the moment of inertia of a disc is

$$I = 1/2\ mr^2$$

so

$$K.E_{rot} = \frac{1}{2} \times \frac{1}{2} mr^2 \omega^2$$

Therefore,

$$K.E_{rot} = \frac{1}{4} mr^2 \omega^2 \quad \text{.....................} (1)$$

we already know that

$$v = r\omega$$

Thus

$$v^2 = r^2 \omega^2$$

Putting this value in equation (1) we get

$$k.E_{rot} = \frac{1}{4} mv^2 \quad \text{.....................} (2)$$

HOOP:- Similarly for a hoop, the moment of inertia is

$$I = mr^2$$

then

$$K.E_{rot} = \frac{1}{2} I\omega^2$$

or

$$K.E_{rot} = \frac{1}{2} mr^2\omega^2 \qquad (mr^2 = I)$$

But

$$r^2\omega^2 = v$$

$$K.E_{rot} = \frac{1}{2} mv^2 \quad \text{..................} (3)$$

When both start moving down an inclined plane of height 'h', their motion consists of both rotational and translational motions as in fig (5.15). If no energy is lost in friction, the total kinetic energy of the disc or hoop on reaching the bottom of the incline must be equal to the potential energy at the top.

$$P.E. = K.E._{Translational} + K.E._{rotational} \quad \text{..............} (4)$$

or

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 \quad \text{......................} (5)$$

where ω and v are the angular and linear speeds at bottom, m is the mass of the rolling body.

For DISC:-

As we know from equation (2)

$$K.E\ rot = \frac{1}{2} I\omega^2 = \frac{1}{4} mv^2$$

Putting it in equation (5), we get

$$mgh = \frac{1}{2} mv^2 + \frac{1}{4} mv^2$$

$$= \frac{2mv^2 + mv^2}{4} = \frac{3\ mv^2}{4}$$

or

$$mgh = \frac{3mv^2}{4}$$

or

$$v^2 = \frac{4gh}{3}$$

Hence,

$$\boxed{v = \sqrt{4gh/3}} \quad \text{.....................} (6)$$

Equation (6) shows the formula for the velocity of disc on reaching the bottom of the inclined plane.

For Hoop:-

As we know from equation (3)

$$K.E_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2} mv^2$$

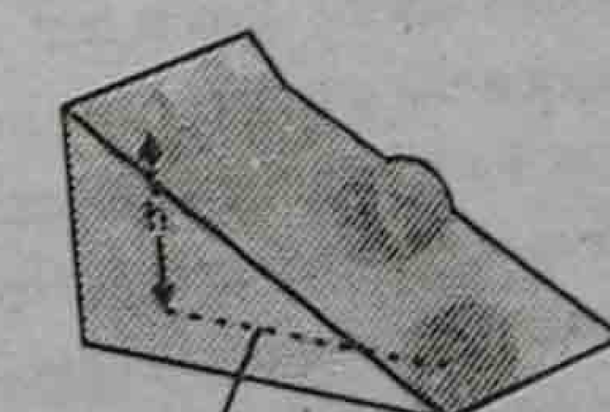


Fig. 5.15

Putting it in equation (5) we get

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} mv^2$$

or

$$mgh = \frac{2\ mv^2}{2}$$

$$gh = v^2$$

or $\boxed{v=\sqrt{gh}}$ .... (7)

Equation (7) shows the velocity of the hoop on reaching the bottom of the inclined plane.

EXAMPLE:-5.6:- A disc without slipping rolls down a hill of height 10.0 m. If the disc starts from rest at the top of the hill, what is its speed at the bottom?

SOLUTION:-

DATA:-

Height of hill = h = 10.0 m

TO FIND:-

speed of disc at bottom = v = ?

FORMULA:-

$$v=\sqrt{4gh/3}$$

CALCULATIONS:-

Using the formula,

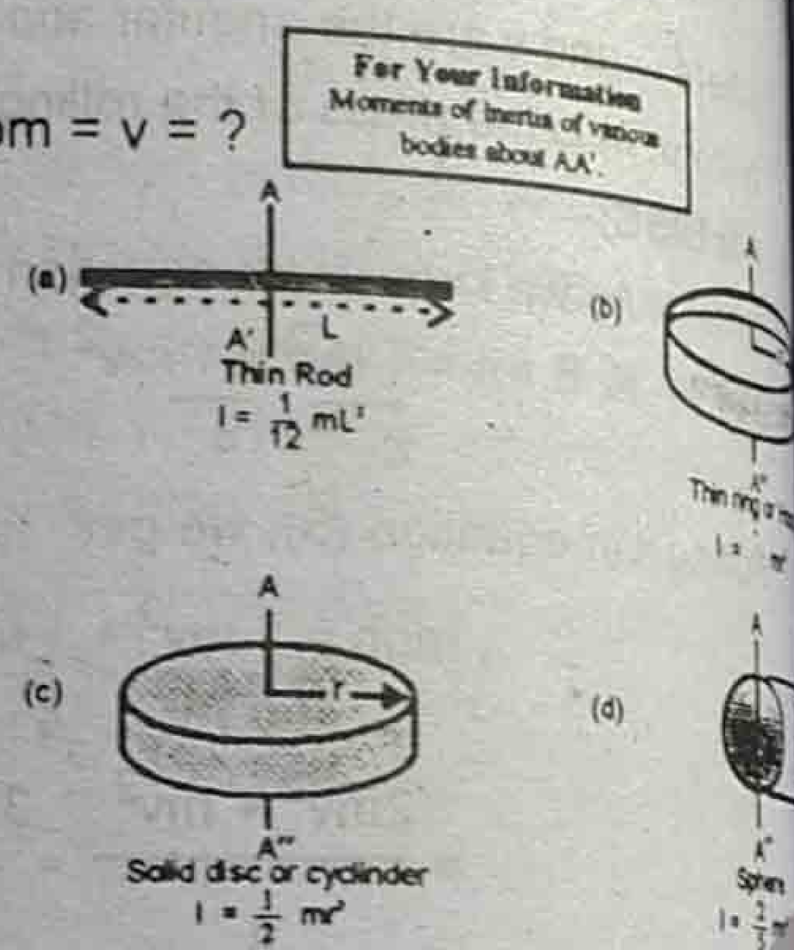
$$v=\sqrt{4gh/3}$$

Putting the values, we have

$$v=\sqrt{\frac{4\times 9.8\times 10}{3}}$$

or $v=\sqrt{392}=11.4\ ms^{-1}$

Hence $\boxed{v = 11.4\ ms^{-1}}$ **Ans.**



NOTE:- For Your Knowlege:-

(i) As the wheel rotates, it has both rotational and translational kinetic energy.

(ii) As the sphere rolls to the bottom of the incline, the gravitational potential energy is changed to kinetic energy of rotation and translation.

5.10.ARTIFICIAL SATELLITES (مصنوعی سیارے)

A man made satellite is called the artifical satellite. The artificial satellites are the objects that orbit (مدار میں چکر لگانا) around the earth due to gravity. They are put into the orbits by rockets and are held in orbits by gravitational pull of Earth. These satellites are launched (چھوڑنا) from the Earth so as to move around it. A number of rockets are fired from the satellite at proper times to establish the satellite in the desired orbit. Once the satellite is placed in the desired orbit with the correct speed for that orbit, it will continue to move in that orbit under the gravitational attraction of the earth.

The satellites which are near the Earth have acceleration 9.8 ms^{-2} towards the centre of the Earth. If they do not have this acceleration, they

would fly off in a straight line tangent to the Earth. The satellites orbiting around the earth may have circular orbits or elliptical orbits.

DETERMINATION OF MINIMUM (OR CRITICAL) VELOCITY:-

When the satellite is moving in a circle, it has an acceleration

$$a_c=\frac{v^2}{r} \quad \text{................ (1)}$$

where v is orbital velocity of the satellite

and r is orbital radius

In a circular orbit around the Earth, the centripetal acceleration supplied by gravity can be found as follows:

As we know that the motion of the satellite in a circle is due to centripetal force. But this centripetal force is provided ([illegible]) by the force of gravity acting towards the centre of Earth. Thus,

$$F=\frac{mv2}{R}$$

but $=F = W = mg$

$\therefore\ mg=\frac{mv^2}{R}$

or $g=\frac{v^2}{R}$ .... (2)

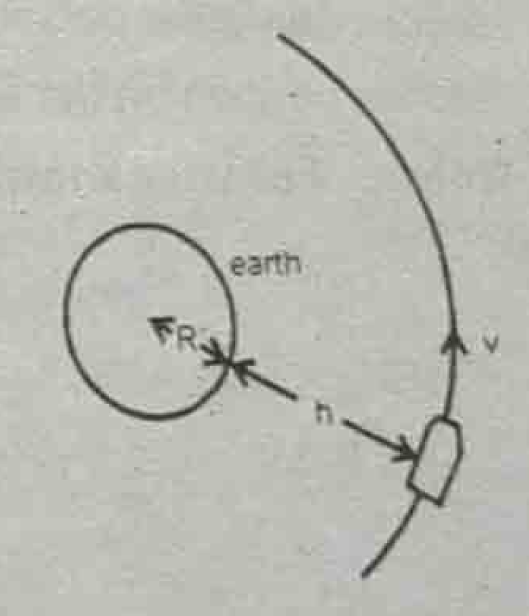


Fig. 5.16

Where v is the orbital velocity and R is the radius of the earth (6400 km). From equation (2) we get

$$v^2=gR$$

or $v=\sqrt{gR}$ .... (3)

Putting the values, $g = 9.8\ ms^{-2}$ and

$R = 6.4\times 106$ m in equation (3), we have

$$v=\sqrt{9.8\times 6.4\times 10^6}$$

$$=7.9\times 103\ ms^{-1}$$

or $\boxed{v = 7.9\ kms^{-1}}$

This is the minimum velocity necessary to put a satellite into the orbit and called critical velocity.

CALCULATION OF TIME PERIOD:-

The time period T is given by

$$T=\frac{2\pi R}{v} \quad \text{.............. (4)}$$

(s = vt)

S = circumference = $2\pi R$

$\therefore\ 2\pi R = vT$

or $T = 2\pi R/v$

Putting the values of R, v we get

$$T=\frac{2\times 3.14\times 6400km}{7.9\ kms^{-1}}$$

= 5060 S = 84 min approx.

CONCLUSION:-

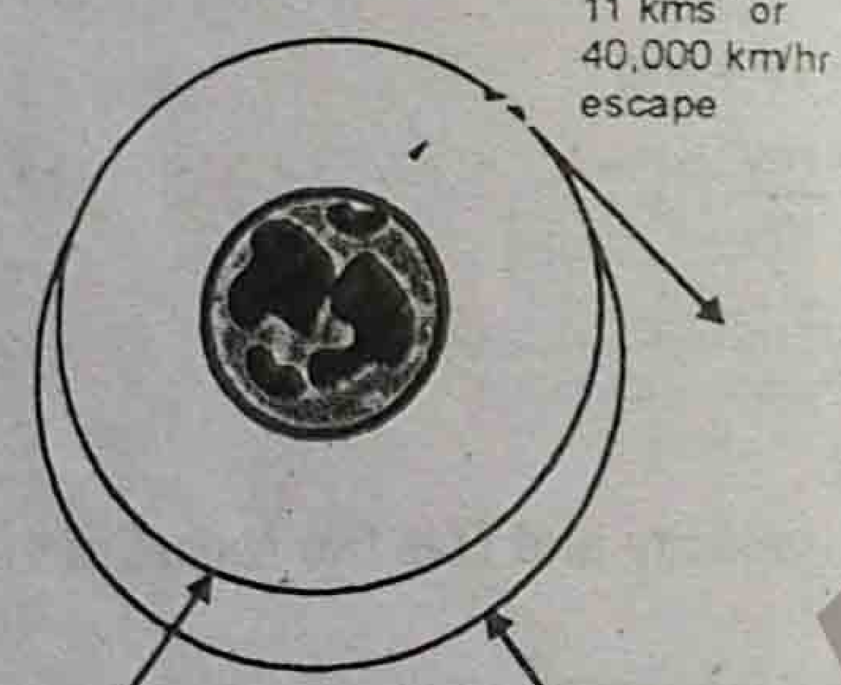
It is concluded that higher the satellite or longer the radius of the orbit lesser (slower) is the speed and longer it will take to complete one revolution around the Earth.

NOTE:- If a satellite in circular orbit is at a large distance `h' above the Earth's surface, then the gravitational acceleration decreases inversely as the square of the distance from the centre of the Earth (fig 5.16)

Global Positioning system:-

When orbiting satillites orbit near the Earth at a hight of 400 Km, then twenty four such satellites form the Global positioning system. An airline pilot, sailor or an other person can now use a pocket size instrument or mobile phone find his position on the Earth's surface to within 10 m accuracy.

Note:- For your knowledge:-



(ii) The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

5.11. REAL AND APPARENT WEIGHT

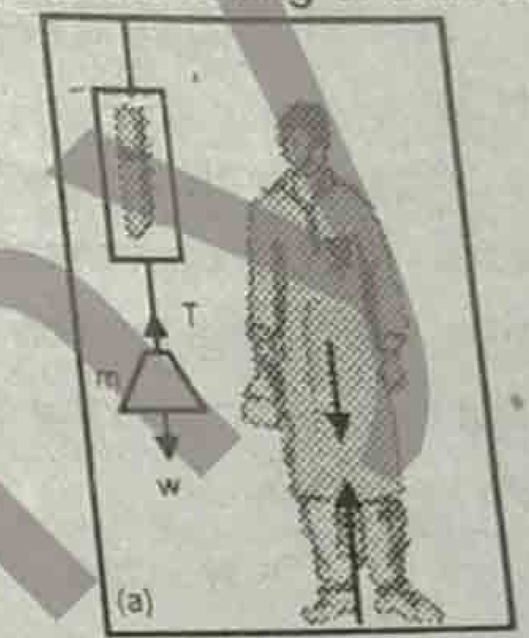
All the objects (اجسام) in a spaceship (خلائی جہاز) revolving around the Earth are in the state of weightlessness (بے وزنی کی حالت). First of all you should know the difference between real and apparant weight.

The gravitational pull of earth acting on an object is called the weight of the object. Similarly, the weight of an object on the surface of the moon is taken to be the gravitational pull of the Moon on the object. The real weight of the object is the value of the gravitational pull when the object lies on the surface of Earth. Usually (عام طور پر) the weight is directly measured by using a balance.

The weight appears to be changed if the object lies(اوپر) in a lift moving up and down with an acceleration. Such a weight is called apparant (ظاہری) weight.

Let us consider an object of mass `m' suspended (لٹکا) from a spring balance by means of a string and spring balance is attached to the ceiling of a lift as shown in fig 5.17(a)

The tension (تناؤ) in the string is always equal in magnitude to the weight of the body. The reading of the spring balance indicates (ظاہر کرتا) the tension in the string. The weight (w =mg) of the object is acting downward. The reading of the spring balance is denoted by `w' and is called apparant weight of the object. Its value depends upon the acceleration of the lift. It should be noted that state of rest or of motion of the object is the same as that of lift.



at rest
$a = 0$
$T = w$

Fig. 5.17(a)

Case 1

(i) **WHEN LIFT IS AT REST:-**

According to Newton's second law of motion, the resultant force on the object is zero. So the acceleration of the object is zero i.e. $a = 0$.

Let `w' be the gravitational force acting on object and T be the tension, then we have

$$T - w = ma$$

as $a = 0$

$\therefore$ $T - w = 0$

or $T = w$ .... (1)

Result:- Thus, the apparant weight of the object is equal to its real weight, according to the observer (مشاہدہ کرنے والے آدمی) inside the lift.

Case II:-

(ii) **WHEN THE LIFT IS MOVING UPWARD WITH AN ACCELERATION `a':-**

Now suppose, the lift moves upward with an acceleration `a' as shown in fig (b). It means the that upward force of tension T on the body is greater than the downward force, weight `w' of the object . Thus,

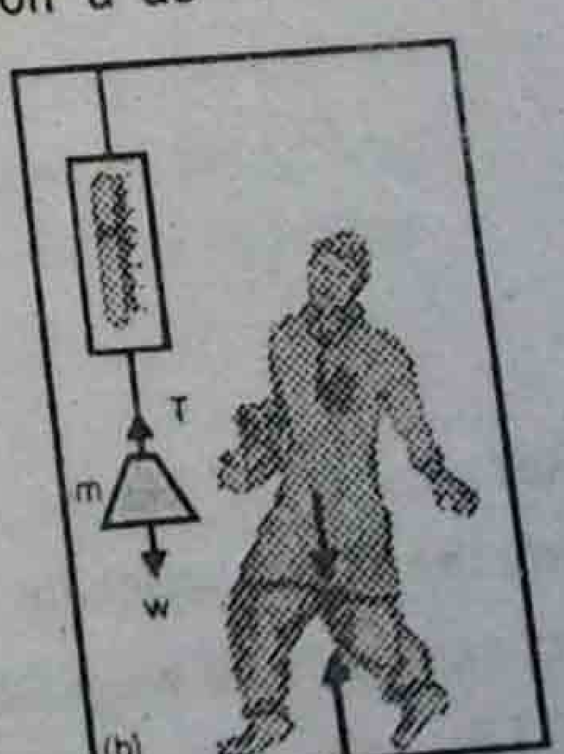
The net force acting on the object = $T - w = F$

But according to Newton's second law of motion

$F = ma$

$\therefore$ $T - w = ma$

or $T = w + ma$ .... (2)



acceleration downward
$w - T = ma$
$T = w - ma$

Fig. 5.17(b)

Result;- This relation shows that the object
ma (i. e. apparant weight has increased by an amount ma).
will weigh more than its real weight by an amount

Case III

(iii) WHEN THE LIFT IS MOVING DOWNWARD WITH UNIFORM ACCELERATION `a'

When the lift alongwith weight is moving downward (fig 5.17 c). It means that the weight (w) is greater than the tension `T' on the object.

∴ The net force acting on the object = w - T = Fnet

or $w - T = ma$ (F = ma)

or $T = w - ma$ .... (3)

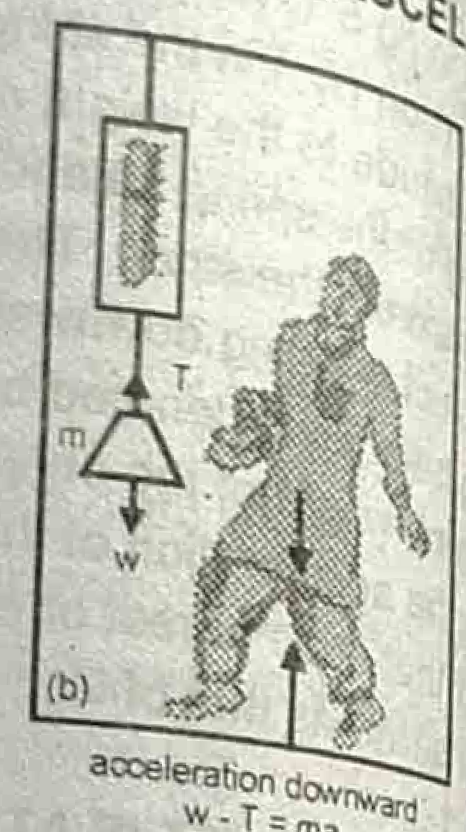


Fig. 5.17(b)

Result:- It is clear from equation (3) that the tension in the string, which is the scale reading, is less than `w' by an amount `ma'. To a person in the accelerating lift, the object appears to weigh less than the real weight `w'. Its apparant weight is then w - ma i.e. it has decrcased by an amount `ma'.

Case IV

(iv) WHEN THE LIFT IS FALLING FREELY UNDER GRAVITY:-

Now we suppose that the lift is falling freely under gravity. Then

$$a = g$$

But $F_{net} = w - T$

or $ma = w - T$ (F = ma)

or $T = w - ma$

As $a = g$

∴ $T = w - mg$

or $T = mg - mg$ (w = mg)

$T = 0$

But $T = w$

∴ $T = w = 0$

Result:- Hence, the spring balance will show zero reading and the man inside the lift will conclude that the apparant weight of the object is zero or the object seems to be weightless. It is the state of weightlessnesss.

NOTE:- You must know

Your apparent weight differs from your true weight when the velocity of the elevator (lift) changes at the start and end of a ride, not during the rest of the

5.12. WEIGHTLESSNESS IN SATELLITES (مصنوعی سیاروں میں بے وزنی) AND GRAVITY FREE SYSTEM:-

When a satellite is falling freely in space, it is attracted towards the centre of Earth at an acceleration a = g. Therefore, the satellites is similar to a lift moving down with an acceleration `g'. Thus, everything inside the satellite will appear to be weightless.

As Earth's satellite is free falling object. Its motion is similar to motion of a projectile. When a projectile is thrown at larger speeds, then during its free fall to the Earth, the curvature (خم۔ موڑ) of the path decreases with increasing horizontal speeds as shown in fig.

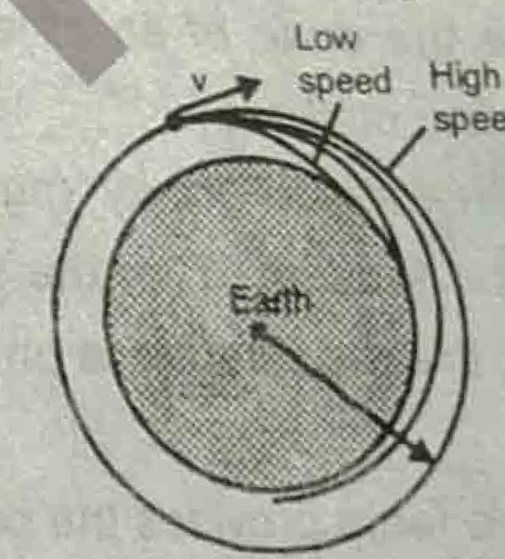


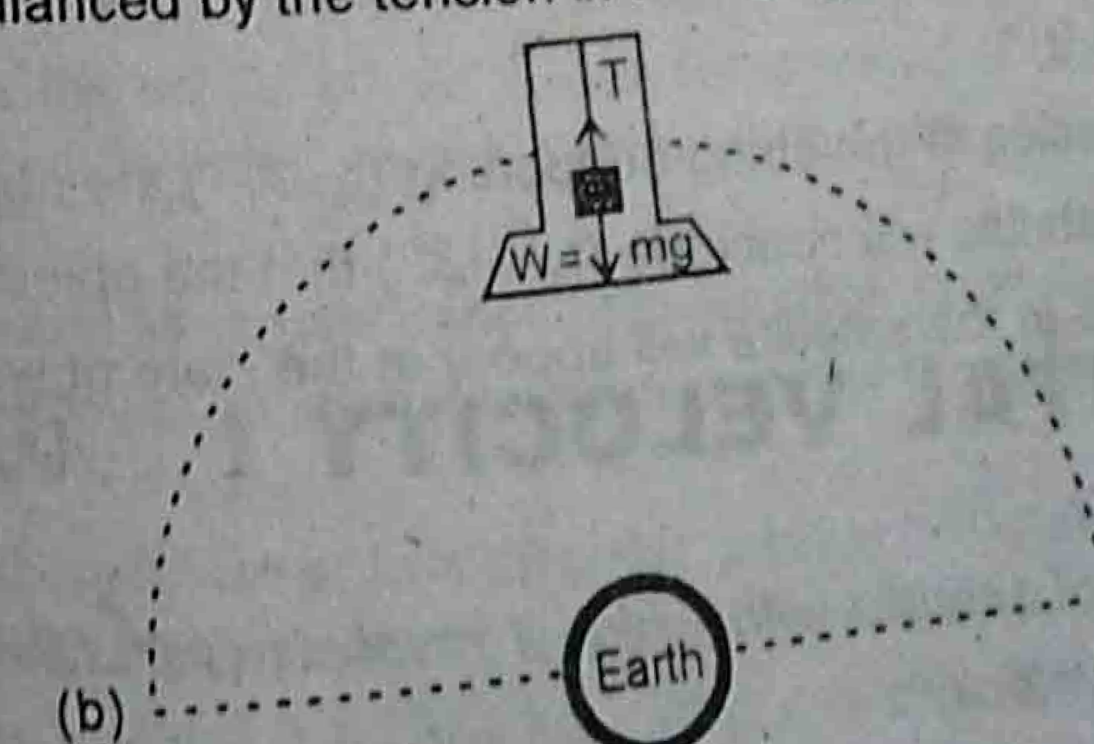
Fig. 5.18

If the projectile is throuwn at very high speed parallel to the Earth, the curvature of its path will match (equal) the curvature of the earth as shown in the fig 5.18. In this case so spaceship (خلائی جہاز) or satellite will start revolving (چکر) around the Earth. This type of projectile is called spaceship or satellite

The spaceship is accelerating towards the centre of the Earth at all times because it circles round the Earth. Its radial acceleration is simply `g', the free fall acceleration. In fact, the spaceship is falling towards the centre of the Earth at all times, but the curvature of Earth prevents (روکنا) the spaceship from hitting the surface of Earth. Since the spaceship is like a free fall object, therefore all the objects inside it appear to be weightless. Thus, no force is preventing the objects from falling in the same frame of reference of the spaceship or satellites. Such a system is called gravity free system.

MATHEMATICAL PROOF:-

Let us consider an object of mass `m' suspended (لٹکا ہوا) by a string (دھاگہ), from the ceiling of a satellite. When a satellite is at rest, the force of gravity (mg) on the object is balanced by the tension in the string as shown in the fig below:



If the statellite is orbiting (گھوم رہا ہے) around the Earth in a circular orbit with a constant speed `v', the only force acting on it is that of gravity. The force of gravity towards the centre of Earth provides the necessary centripetal force for the satellite. Thus,

$$M_s g = \frac{M_s v^2}{r}$$

Where M_s = mass of satellite

r = radius of orbit

or $g = v^2/r$ .... (1)

Now take the case of an object suspended in the satellite. The forces of gravity acting on it are

(i) The force of gravity `mg' pulling it towards the centre of Earth.

(ii) The tension `T' in the string acting away from the centre of Earth.

Therefore, the resultant force on the object is

$$F = mg - T$$

This force provides the centripetal force $\frac{mv^2}{r}$ to keep it in circular orbit.

$$\therefore \quad mg - T = mv^2/r$$

Dividing both sides by `m' we get

$$g - \frac{T}{m} = \frac{v^2}{r} \quad \text{.............................} \ (2)$$

putting the value of $g = v^2/r$ from equation (1) in equation (2), we get

$$g - \frac{T}{m} = g$$

or

$$\frac{-T}{m} = g - g$$

or

$$\frac{T}{m} = 0$$

But `m' cannot be zero, therefore

$$T = 0$$

It means that the tension in the string supporting (سہارا دینا) the object is zero. So an observer in the satellite will conclude (نتیجہ اخذ کرنا) that the object is weightless. Hence, all the objects in the satellite will appear in the state of weightlessness.

5.13.ORBITAL VELOCITY (محوری رفتار)

DEFINITION:-

The velocity of satellite with which it rovolves (چکر لگانا۔ گھومنا) round the Earth is called orbital velocity.

The Earth and some other planets revolve around the sun in nearly circular paths. Similarly, the artifical satellites launched (بھیجنا) by men also adopt (اختیار کرنا) nearly the circular path around the Earth. This type of motion is called Orbital motion.

EXPRESSION FOR ORBITAL VELOCITY:-

Consider a satellite going round the Earth in a circular path as shown in the fig. 5.19. Let the mass of the satellite be `m' and `v' be its velocity. Suppose the mass of the Earth is `M' and r the radius of the orbit. We know that the satellite can continue its motion in the circular orbit if Earth's gravitational force acting on it provides the necessary centripetal force. In this case

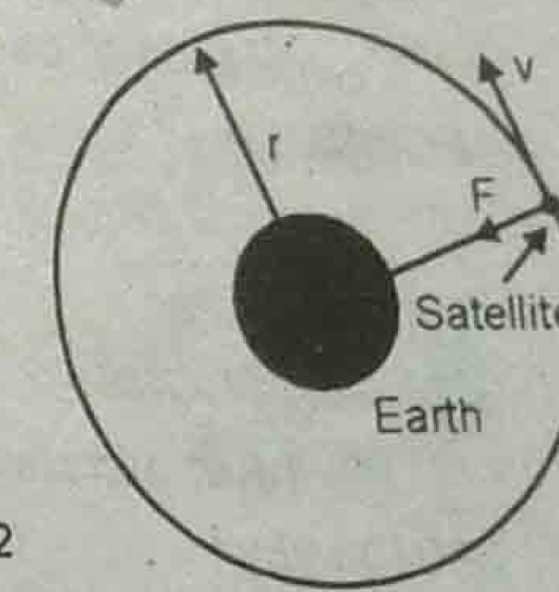


Fig. 5.19

The centripetal force on the satellite = $\frac{m_s v^2}{r}$

and

the gravitational force on it = $\frac{G M ms}{r^2}$

Hence, equating the gravitational force to the required centripetal force, we can write

$$G \frac{m_s M}{r^2} = \frac{m_s v^2}{r}$$

or $v^2 = \frac{GM}{r}$

or $v = \sqrt{\frac{GM}{r}}$ .... (1)

This equation (1) shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus any satellite orbiting at distance `r' from Earth's centre must have the orbital speed $\sqrt{GM/r}$. If the speed of the satellite is less than this value, it will not be able to revolve around the Earth. Thus, it falls back to the Earth.

Since G and M remain constant in equation (1) therefore orbital speed of a satellite is inversely proportional to square root of the radius of the orbit.

EXAMPLE 5.7.

An Earth satellite is in a circular orbit at a distance of 384,000 km from the Earth's surface, what is the period of one revolution in days? Take mass of the Earht M = 6.0 × 1024 kg and the radius R = 6400 km.

SOLUTION:-

DATA:-

Height of satellite from Earth = h = 384,000 km

Mass of Earth = m = 6.0×10^{24} kg

Radius of Earth = R= 6400 km

TO FIND;-

Period of one revolution = T= ?

FORMULA:-

$$T = \frac{2\pi R}{v}$$

CALCULATIONS:-

Total distance of satellite from Earth = R + h = 6400 + 384000 = 390400 km.

using the formula.

$$v = \sqrt{\frac{GM}{r}}$$

Puting the values, we get

$$v = \sqrt{\frac{6.67\times10^{-11} \times 6\times10^{24}}{390400}}$$

$$= \sqrt{\frac{6.67 \times 6 \times 10^{11}}{3904}}$$

$$= 1.01\ kmS^{-1}$$

Also we know that

$$T = \frac{2\pi R}{v}$$

Putting the values, we get

$$T = 2\times3.14\times390400 \times \frac{1}{1.01} \times \frac{1\ day}{60\times60\ 24s}$$

Hence, $\boxed{T = 27.5\ days}$ **Ans.**

RESULT:- Period of one revolution is 27.5 days

5.14. ARTIFICIAL GRAVITY (کشش ثقل)

we know that the astronaut (خلاباز۔مسافر) orbiting around the Earth is in the state of weightlessness. There will be no force pressing him to any side of the spacecraft or spaceship (خلائی جہاز). If the spaceship (or spacecraft) is to stay in the orbit (مدار) for a longer time, this weightlessness creates a lot of (بہت زیادہ) problems for the astronaut while performing his duty in the spaceship. To

overcome (قابو پانا) this difficulty, an artificial gravity is created inside the spaceship (satellite) so that the astronauts may perform their experiments efficiently (موثر طریقے) and in normal manner as they do in the Earth's gravity. This artificial gravity is achieved (حاصل کرنا) by rotating the spaceship around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the floor of the spaceship in the same way as on the Earth. This occurs due to the artifical gravity.

EXPRESSION FOR FREQUENCY:-

Consider a ring shaped space station as shown in fig 5.20. The outer radius of the space ship is 'R' and it rotates arount its own central axis with a rim speed 'n'. The centripetal acceleration experienced (محسوس کی ہوئی) by a point on the outer rim (کنارا۔ پہیہ کا گھیرا) is given by

$$a_c = v^2/R \quad .... (1)$$

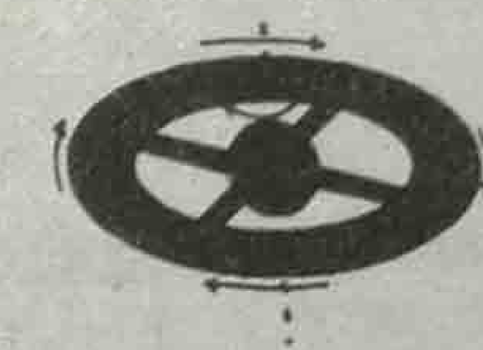


Fig. 5.20

Multiply both sides of equation (1) by 'm', we have

$$ma_c = \frac{mv^2}{R}$$

The astronaut, therefore, exerts (لگاتا ہے) a force of reaction (centripetal force) 'Fr' on the outer rim which is given

$$F_r = ma_c = \frac{mv^2}{R} \quad .... (2)$$

The astronaut is pressed towards the outer rim and exerts a force on the floor of the spaceship in the same manner as on the Earth.

Centripetal acceleration of spaceship is given by

$$a_c = \frac{v^2}{R}$$

But $v = R\omega$

$$a_c = \frac{R^2\omega^2}{R} = R\omega^2 \quad .... (3)$$

where w is angular speed.

But $\omega = 2\pi/t$ $\quad (\omega = \theta/t = 2\pi/t)$

Putting the value of w in equation (3)

$$a_c = \frac{R(2\pi)^2}{t^2}$$

Hence

$$a_c = \frac{R4\pi^2}{t^2} \quad .... (4)$$

Let 't' be time for one revolution of the satellite or spaceship and 'f' be the frequency of rotation, Thun,

$$t = 1/f \quad \text{or} \quad f = 1/t$$

Now putting the value of 't' in equation (4) we get

$$a_c = \frac{R4\pi^2}{(1/f)^2} = R4\pi^2 f^2$$

or

$$f^2 = \frac{a_c}{4\pi^2 R}$$

or

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}} \quad \text{.............. (5)}$$

We know that in case of satellite orbiting round the earth, the force of gravity provides the required centripetal acceleration, in this case reverse (الٹ) is true i.e the centripetal force produces the force of gravity

$$\frac{mv^2}{R} = mg$$

or

$$v^2/R = g$$

$$\therefore \quad a_c = g$$

Hence, equation (5) becomes

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}} \quad \text{.............. (6)} \qquad (a_c = g)$$

At this frequency artificial gravity equal to the gravity of Earth will be provided (مہیا کی جاتی) to the occupants (باشندے) of the spaceship. Now, the astronauts do not feel any difficulty in perfoming their duties.

NOTE:- For Your knowledge:-

(1) In 1964, at a height of 100 km above Hawali Island with a speed of 29000 km h^{-1} Bruce Mccandless stepped into space from a space shuttle and became the first human satellite of the Earth.

(2) The surface of the rotating spaceship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

5.15. GEOSTATIONARY ORBITS

DEFINITION:-

The orbit of satellite in which its motion is synchronized (وقت کی مطابقت رکھنا، ہم وقت کرنا) with the rotation of Earth, is called geostationary orbit.

OR

A geo-stationary ordit is that in which the period of rotation of the satellite around the earth is exactly equal to the period of rotation of the Earth about its axis.

EXPLANATION:-

An important and useful example of satellite motion is geo-synchronous or geo-stationary satellite. In such type of satellite the orbital motion takes place (واقع ہونا) at the same time with the rotation of the Earth. In

this way the geo-stationary satellite remains always over the same point on the equator (خطِ استوا) as the Earth spins (rotates) on its axis. In other words, when the satellite is in geo-stationary state, the satellite appears to be stationary with respect to the point on the Earth from which it is seen.

EXPRESSION FOR THE ORBITAL RADIUS OF GEOSTATIONARY SATELLITE:-

As we know that the orbital speed of a satellite is given by

$$v = \sqrt{GM/r} \quad \text{.................. (1)}$$

where
M = Mass of the Earth
r = Distance of satellite from Earth
G = Gravitational constant

But this speed `v' is equal to the average speed of the satellite in one day i.e.

$$v = S/t = 2\pi r/t \quad \text{.................. (2)} \qquad (\text{circumerence} = s = 2\pi r)$$

where 't' is the period of revolution of the satellite, that is equal to one day. This means that the satellite must move in one complete orbit in a time of exactly one day.

As the Earth rotates in one day and the satellite will revolve (چکر لگانا) around the Earth in one day, so the satellite at position `A' will always stay over the same point A on the Earth, as shown in fing 5.21.

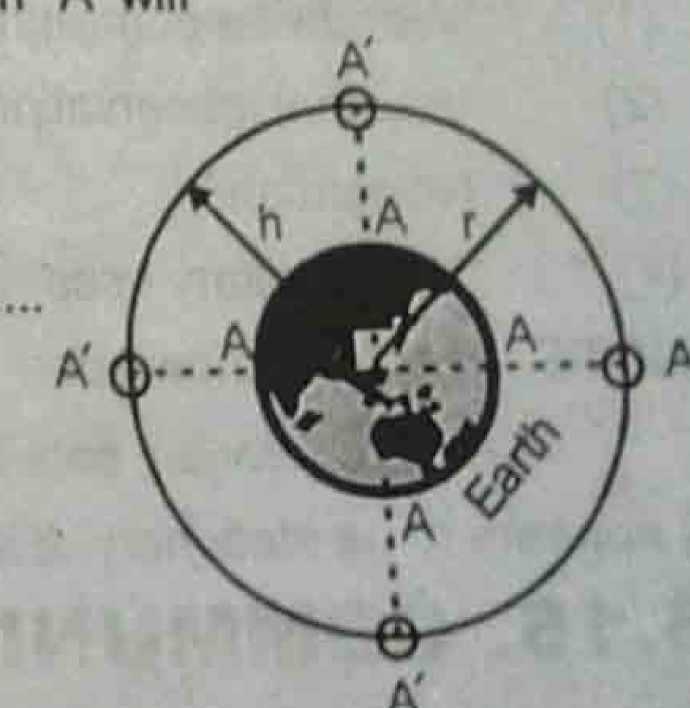


Fig. 5.21

Equating equations (1) and (2) we have

$$\frac{2\pi r}{t} = \sqrt{\frac{GM}{r}} \quad \text{..................}$$

squaring both sides equations (3) we get

$$\frac{4\pi^2 r^2}{t^2} = \frac{GM}{r}$$

or

$$r^3 = \frac{GMt^2}{4\pi^2}$$

or

$$r = \left[\frac{GMt^2}{4\pi^2}\right]^{1/3} \quad \text{.............. (4)}$$

knowing the values of G, M, t, we put it in equation (3)

$G = 6.67 \times 10^{-11}$ N-m kg^{-2}

$M = 6 \times 10^{24}$ kg

$t = 1$ day $= 24$ hours $= 24 \times 60 \times 60s = 86400$ s

$$r = \left[\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (86400)^2}{4 \times (3.14)^2}\right]^{1/3}$$

$$= (7.575 \times 10^{22})^{1/2}$$

or

$$r = 0.4236 \times 10^8 \text{ m}$$

or $r = 4.23 \times 10^4 \times 10^3$ m

Hence $\boxed{r = 4.23 \times 10^4 \text{ km}}$ .... (6)

Which is the orbital radius measured from the centre of the Earth, for a geo-stationary satellite.

Height of Geo-stationary Satellite From the Surface of Earht:-

Let h be height of satellite above the surface of Earth, then

$$r = R + h$$

or $$h = r - R$$

Putting the values of r and R, we get

$$h = 4.23 \times 10^4 - 6400$$

$$= 42300 - 6400 = 35900 \text{ km}$$

$$h = 36000 \text{ km approxi} \quad \text{......................} \quad (7)$$

This equation (7) shows that the height at which the satellite will always stay directly above a point on the surface of the Earth is 36000 km.

USES OF GEO - STATIONARY SATELLITES:-

Such satillites are useful for the following purposes.

(1) Worldwide communication
(2) Weather observations
(3) Navigation
(4) Other military uses

NOTE:- For Your knowlede:-

A geostationary satellite orbits the Earth once per day over the equator so it appears to be stationary. It is used now for international communication.

5.16. COMMUNICATION SATELLITES

Now - a days, the geo - stationary satellites are being used in order to supply the informations througout the world. A communication system can be set up (قائم کرنا) by placing several geo-stationary satellites in orbit over different points on the surface of the Earth. One such satellite covers 120 of longitude (طول بلد). The whole of populated earth's surface can be covered by three correctly positioned satellites as shown in fig 5.22. Since these geo-stationary satellites seem to rotate over one place on the Earth, so continuous communication with any place on the surface of the Earth can be made.

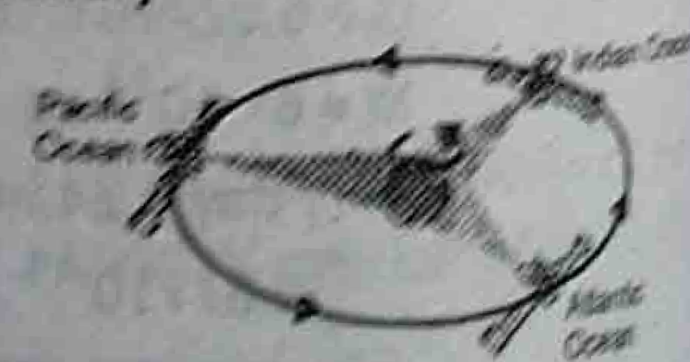


Fig. 5.22

Microwaves are used as the carrier of communication signals because they travel in a narrow beam, in straight line

and pass easily through the atmosphere of the Earth. Large solar cell panels are fitted on the satellites in order to (کی غرض سے) supply the energy needed to amplify (بڑھانا) and retransmit (بھیجنا) the signals.

There are over 200 Earth stations which transmit signals to satellites and also receive signals via satellites from other countries. These signals can also be picked up from satellite (حاصل کرنا) by using a dish antenna on the roofs of houses. A largest satellite system is managed by126 countries, that is called International Telecommunication Satellite Organization . An INTELSAT VI satellite is shown in fig 5.23

Fig. 5.23

It operates at microwave frequencies of 4,6, 11 and 14 GHZ and has a capacity of 30,000 two wave telephone circuits plus three T. V. channels.

EXAMPLE 5.8:-

Radio and TV signals bounce from a synchronous satellite. This satellite circles the Earth once in 24 hours. So if the satellite circles eastward above the equator, it stays over the same spot on the Earth because the Earth is rotating at the same rate. (a) What is the orbital radius for a synchronous satellite (b) What is its speed?

SOLUTION:-

DATA:-

- Mass of the Earth = M = 6.0×1024 kg

Time = t = 24 hours = $24 \times 60 \times 60$ s

Gravitational constant = G = 6.67×10^{-11} $N\text{-}m^2kg^{-2}$

TO FIND:-

(a) Orbital radius for synchronous =?
(b) Speed of satellite = v = ?

FORMULA:-

$$r = \left[\frac{GMt^2}{4\pi^2}\right]^{1/3}$$

CALCULATIONS:-

Using the formula

$$r = \left[\frac{GMt^2}{4\pi^2}\right]^{1/3}$$

Putting the values, we get

$$r = \left[\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (24 \times 600 \times 60)^2}{4 \times (3.14)^2}\right]^{1/3}$$

On simplification,we get

$\boxed{r = 4.23 \times 10^7 \text{ m}}$ Ans.

(b) As we know that the speed of a satellite is expressed by the formula

$$v = 2\pi r/t$$

Putting the values of r and t in the formula, we get

$$v = \frac{2\times 3.14\times 4.23\times 10^7}{24\times 60\times 60}$$

$$= \frac{26.5644\times 10^7}{86400} = \frac{26.56\times 10^5}{864}$$

$$= .0307\times 10^5 \text{ ms}^{-1} = 3.1\times 10^3 \text{ ms}^{-1}$$

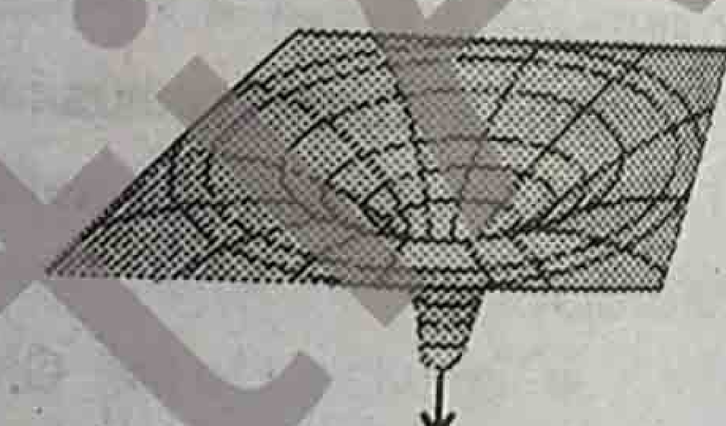
$\boxed{v = 3.1 \text{ kms}^{-1}}$ Ans.

5.17. NEWTON'S AND EINSTEIN'S VIEWS OF GRAVITATION:-

According to Newton, the gravitation is the natural (real) property of matter that every particle of matter attracts every other particle with a force which is directly proportional to the product of their masses and is inversely proportional to the square of the distance between them.

According to Einstein's theory, space (جگہ) time is curved near massive (بھاری) bodies. In order to (کی غرض سے) observe (مشاہدہ کرنا) it, we might think of space as a thin rubber sheet, if a heavy weight is hung (لٹکانا) from it, it curves or dents (بل کھانا۔ خم پڑجانا) as shown in fig 5.24. The weight refers (حوالہ دینا) to a huge mass that causes (سبب ہونا) space itself to curve.The heavier weight (mass), the greater the curve or dent (خم).

In Einstein's theory, we do not talk about the force of gravity acting on bodies, instead (کے بجائے) we say that bodies and light rays move along straight lines in curved space time. Thus,a body moving or at rest near a heavy mass (fig 5.24) would follow a straight line (geodestic) towards the great (or heavy) mass.



Weight

Einstein's theory gives us a physical picture of how gravity works. Newton discovered the inverse square law of gravity but could not explain the reason for obeying inverse square law. Einstein's theory also says that gravity obeys (or follows) an inverse square law (except in strong gravitational field), but it fully explains the reason for obeying the inverse square law. That is why Einstein's theory is better than Newton's views (theory).

Einstein concluded(نتیجہ اخذ کرنا) that if gravity and acceleration are exactly equivalent (مساوی), gravity must bend light by a definite (خاص) amount that

could be calculated. Newton's theory, based on the idea of light as a stream (ریلا) of tiny (بہت چھوٹے) particles, also suggested that a light beam (کرن) would be deflected (راستہ بدلنا۔ مڑنا) by gravity .But in Einstein's theory, the deflection of light is twice (double) as great as it is according to Newton's theory.

When the bending of straight starlight caused by the gravity of the sun was measured during a solar eclipse (سورج گرہن) in 1919.This measurement has matched Einstein's views rather than (کی بجائے) Newton's. Therefore, Einstein's theory was considered as scientific triumph (فتح).

QUESTIONS WITH ANSWERS

Q.5.1:- Explain the difference between tangential velocity and the angular velocity. If one of these is given for a wheel of known radius, How will you find the other?

Ans:- **Tangential Velocity:-**

When an object (جسم) moves along a circle with constant speed, the magnitude of linear velocity of the object remains constant but its direction changes continuously (لگاتار) from point to point. The direction of linear velocity is always along the tangent (مماس) on any point of the circle. This linear velocity is known as the tangential velocity. It is denoted by v_t

Angualr velocity:-

The angular velocity is rate of change of angular displacement of an object moving along a circle. It is vector quantity and denoted by `ω'. Its direction is found out by right hand rule stated as follows:-

Curl (لپیٹنا) the fingers of right hand around the rotation axis in the direction of rotation, then the thumb points (اشارہ کرنا) towards the direction of angular velocity $\vec{\omega}$. $\vec{\omega}$ is represented by a line drawn parallel to the axis of rotation.

The magnitude of tangent velocity of an object moving in a circle is given by the product of the distance of the object from the axis of rotation and the angular speed i.e.

$$v_t = r\omega \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Q:3 knowing the radius of wheel, if one of these is given, the other can be found by using the above relation (1).

Q.5.2 Explain what is meant by centripetal force and why it must be furnished (بہم پہنچانا) to an object if the object is to follow a circular path?

Ans:- When a body moves in a circle with constant speed, the force which

keeps the body moving in the circular path and always directed towards the centre of the circle is called **centripetal force**. The magnitude of the centripetal force is

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Without this force, no body can move in a circular path.

According to Newton's first law of motion, a body can move along a straight line with uniform velocity only if no net force acts on it. For uniform circular motion, it must be under the continuous influence (اثر) of some force that changes the direction of velocity of the body at every instant (لمحہ) and thus produces the acceleration in the body. The centripetal force is always needed if the body is to be maintained (برقرار رکھنا) in its circular path.

Q.5.3. What is moment of inertia? Explain its significance:-

Ans:- Moment of Inertia:- (جمودی معیار) *(D. G. Khan 1990, Lahore 1989, Gujw 1995)*

In case of linear motion, every body opposes the force which is applied to its state of rest or of motion. This property of a body to oppose the accelerating force is called inertia. Similarly all the rotating bodies oppose the torque which is applied to change their state of rotatory motion. *The property of a body to oppose the accelerating torque is known as its moment of inertia.*

Definition:- *It is defined as the product of mass of particle and square of its distance from axis of rotation. It is denoted by I*

$$\therefore \quad I = mr^2$$

If the rigid body is made of n - particles, then different particles are at different distances from axis of rotation. Thus:

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots m_n r_n^2$$

where $m_1, m_2 + \ldots\ldots\ldots\ldots\ldots\ldots$ are masses of n particles at distances $r_1, r_2 + \ldots\ldots\ldots\ldots\ r_n$ from axis of rotation respectively. So, equation (1) can be written as

$$I = {}^n\sum_{i=1} m_i\, r_i^2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Physical Signification:-

For a body in linear motion the acceleration is proportional to the force acting upon the body i.e. the ratio of the force to acceleration is constant. Mathematically it is written as

$$F/a = m \text{ (constant)} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

where 'm' is the mass of the moving body which is a direct measure of its inertia.

Similarly for a body rotating about any axis, the angular acceleration is proportional to the torque acting on the body i.e the ratio of the torque to the angular acceleration is constant. Mathematically, it is expressed as

$$\tau/\alpha = I \text{ (a constant)} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

By comparing (1) and (2) it can be seen the moment of inertia of a rotating body is analogous (مشابہ) to the mass of a body in linear motion. Hence, moment of inertia may be described as rotational mass of a body. That is moment of inertia plays the same role in angular motion as the mass in linear motion.

Q.5.4. What is meant by angular momentum? Explain the law of conservation of angular momentum. (Lahore 2000, Faisalabad 2000, 2001)

Ans:- Angular Momentum:-

The angular momentum of an object is defined as the product of position vector $\vec{r}$ with respect to the axis of rotation and linear momentum $\vec{p}$ of an object. It is denoted by $\vec{L}$.

Mathematically it is written as

$$\vec{L} = \vec{r} \times \vec{p} \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Putting the value of $\vec{p}$ in equation (1) we get

$$\vec{L} = \vec{r} \times (m\vec{v}) \qquad (\vec{p} = m\vec{v})$$

or

$$\vec{L} = m(\vec{r} \times \vec{v})$$

Second Definition:-

The angular momentum is also defined as the product of moment of inertia 'I' and the angular velocity 'ω' of the body. i.e

$$L = I\omega$$

In angular motion, the moment of inertia 'I' plays the same role as the mass 'm' in linear motion. In linear motion, the momentum of a body is mv, therefore the angular momentum of a rotating body is Iω

Law of conservation of Angular Momentum:-

As in the case of linear motion, linear momentum remains constant if no external force acts upon it. Similarly in rotatory motion, angular momentum remains constant if no external torque acts upon it.

Statement:-

The law of conservation of angular momentum states that the angular momentum about any axis of a given rotating body or a system of bodies is constant, it no external torque acts about that axis.

OR

If no external torque acts on a system, the tatal angular momentum remains constant. That is

$$L_{total} = L_1 + L_2 + \ldots\ldots\ldots\ldots = \text{constant}$$

lustration ([illegible]) consider the example of a stone whirled ([illegible]) at the end of a string. If we stop exerting force on the string and allow it to wind ([illegible]) on the finger, the length of the string will go on decreasing while the angular speed of the stone will go on increasing continuously. As no new torque acts on the stone (because force is zero), the angular momentum $I\omega$ will remain constant. When the length of the string decreases, `I' also decreases and for keeping $I\omega$ constant, the angular speed increases.

It can be expressed as

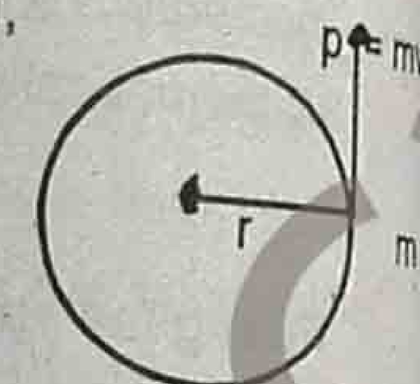
$$L = I_1\omega_1 = I_2\omega_2 = \text{constant}$$

If moment of inertia `I' of body decreases, its angular velocity `ω' increases, so that their product remains constant.

Q.5.5. Show that orbital angular momentum, L_o = mvr

Ans:- According to the definition of angular momentum,

$$\vec{L}_o = \vec{r} \times \vec{p}$$



Its magnitude is given by

$L_o = rp\sin\theta$ .... (1)

where `θ' is the angle between position vector $\vec{r}$ and velocity of linear momentum $\vec{p}$.

Equation (1) can be written as.

$L_o = mrv\sin\theta$ .... (2)

In case of circular orbital motion such as satellite of mass `m' revolving around the Earth in a circle of radius r, the angle between radius `r' and tangential velocity `v' is always 90°.Therefore,

$$L_o = mrv\sin 90^\circ$$

Hence $\boxed{L_o = mrv}$ $(\sin 90^\circ = 1)$

5.6. Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

Ans:- When the satellite is moving in a circle, it has centripetal acceleration

$a_c = v^2/r$ .... (1)

In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity can be found out as

$F = mv^2/R$ .... (2)

(centripetal force is provided by force of gravity)

But $F = w = mg$

$\therefore$ $mg = \dfrac{mv^2}{R}$

or $g = \dfrac{v^2}{R}$

or $v^2 = gR$

Hence $v = \sqrt{gR}$ .... (3)

where R is the radius of Earth = 6400km

$g = 9.8\ ms^{-2}$

Putting these values in equation (3) we get

$v = \sqrt{9.8 \times 6400 \times 1000} = \sqrt{9.8 \times 6.4 \times 10^6}$

$= \sqrt{9.8 \times 6.4} \times 10^3\ ms^{-1}$

$= 7.9 \times 10^3\ ms^{-1}$

$\therefore$ $\boxed{v = 7.9\ kms^{-1}}$

This is the minimum velocity necessary to put a satellite into orbit around the Earth.

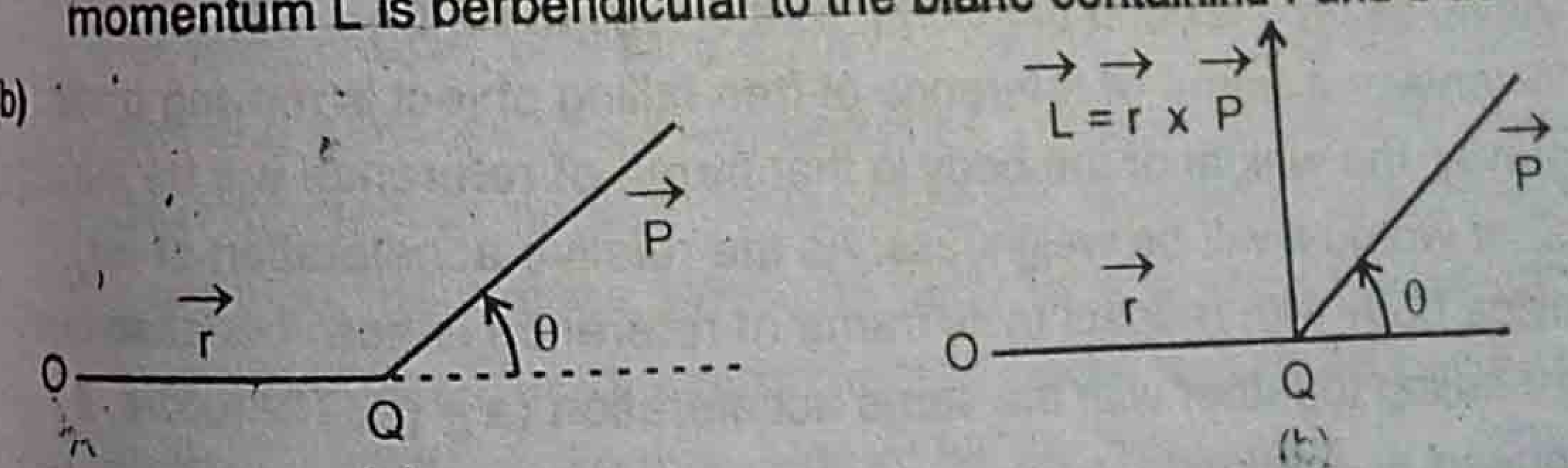
Q.5.7. State the direction of the following vectors in simple situations; angular momentum and angular velocity?

Ans:- Direction of Angular Momentum:-

As we know that the angular momentum is defined by

$$\vec{L} = \vec{r} \times \vec{p}$$

The angular momentum is a vector quantity. The direction of angular momentum $\vec{L}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{p}$ as shown in fig (b)



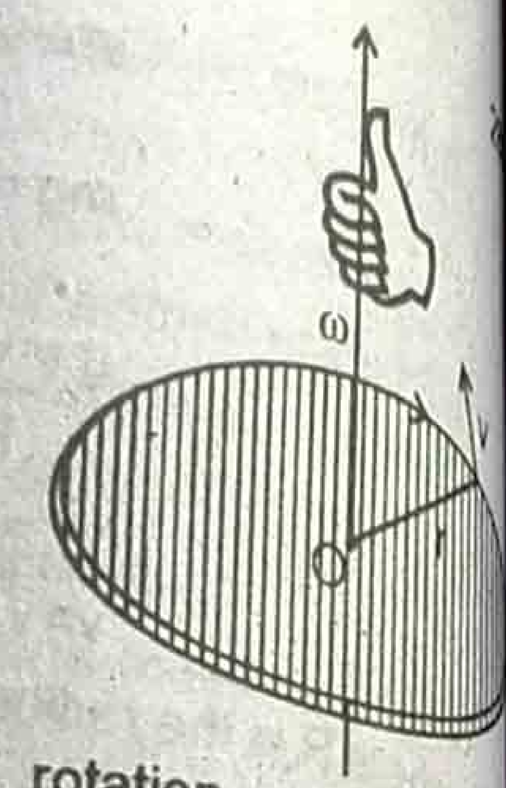
Its direction is determined by right hand rule.

If an object is rotating along certain axis, then direction of $\vec{L}$ will be along axis of rotation. According to right hand rule, for a body having counter clockwise rotation, the angular momentum is directed outward along the axis of rotation.

Direction of Angular velocity:-

Angular velocity is a vector quantity. Its direction also can be found by the right hand rule.

Curl the fingers of the right hand around the rotation axis in the direction of rotation, then thumb points (اشارہ کرنا) towards the direction of angular velocity `w' as shewn in fig.



$\vec{w}$ is represented by a line drawn parallel to the axis of rotation. By the right hand rule for counter (anti) clockwise rotation, the direction of angular velocity is outward along the axis of rotation.

Q.5.8. Explain why an object, orbiting the Earth, is said to be freely falling. Use your explanation to point out why object appear weightless under certain circumstances?

Ans:- We consider an object as artificial satellite which is put into an orbit around the Earth by giving a suitable tangential orbital velocity. The satellite is put into orbit by rocket and is held in an orbit by gravitational pull of Earth. Its centripetal acceleration is equal to the acceleration due to gravity directed towards the centre of Earth. Therefore, a satellite is always falling towards the centre of Earth with an accleration `g'. Hence, the satellite is said to be free falling object. Due to the tangential velocity and downward velocity due to force of gravity, it moves along a curved path. The curvature of this path is such that the Earth curves around by the same amount as the moving object and therfore does not touch the surface of earth. As the object continues to fall during its orbit around the Earth, so it is said to be free falling.
Whenever a frame of reference of free falling object is moving under gravity, the weight of the body in that frame of reference will be zero. In other words it will be weightless. As the relative acceleration of an inside body with respect to its frame of reference is zero because both are falling together with the same acceleration (a = g). Therefore, it appears weightless.
Hence, if the satellite is in free fall, all the objects inside it appear to be weightless.

Q.5.9. When mud(کیچڑ) flies off the tyre of moving bicycle, in what direction does it fly? Explain ***(Sargodha 1988)***

Ans:- The mud will fly off (اُچھل جانا) tangentially along a straight line. When the tyre rotates, a centripetal force acts on the mud which is equal to the adhesive (چپکنے والی) force between the tyre and mud. When

the angular speed of the tyre increases, the centripetal force on the mud also increases. When this centripetal force is greater than the adhesive force, the mud leaves the tyre and fly off tangentially along a straight line due to centrifugal force which is simply the reaction of the centripetal force.

Q.5.10. A disc and a hoop start moving down from the top of an inclined plane at the same time, which one will be moving faster on reaching the ground ? ***(Lahore 1985, Sargodha 1984)***

Ans:- **HOOP:-**
Velocity of the hoop moving down the inclined plane is given by

$$v = \sqrt{gh} \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Disc:-
Velocity of the disc moving down the inclined plane is given by

$$v = \sqrt{4gh/3}$$

or $$v = \sqrt{4/3} \times \sqrt{gh}$$
$$= 1.15 \times \sqrt{gh} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

comparing the equations (1) and (2) we see that velocity of disc is greater than that of hoop.
Hence, the disc will be moving faster on reaching the ground.

Q.5.11. Why does a diver (غوطہ زن) change his body positions before diving in the pool (جوہڑ)

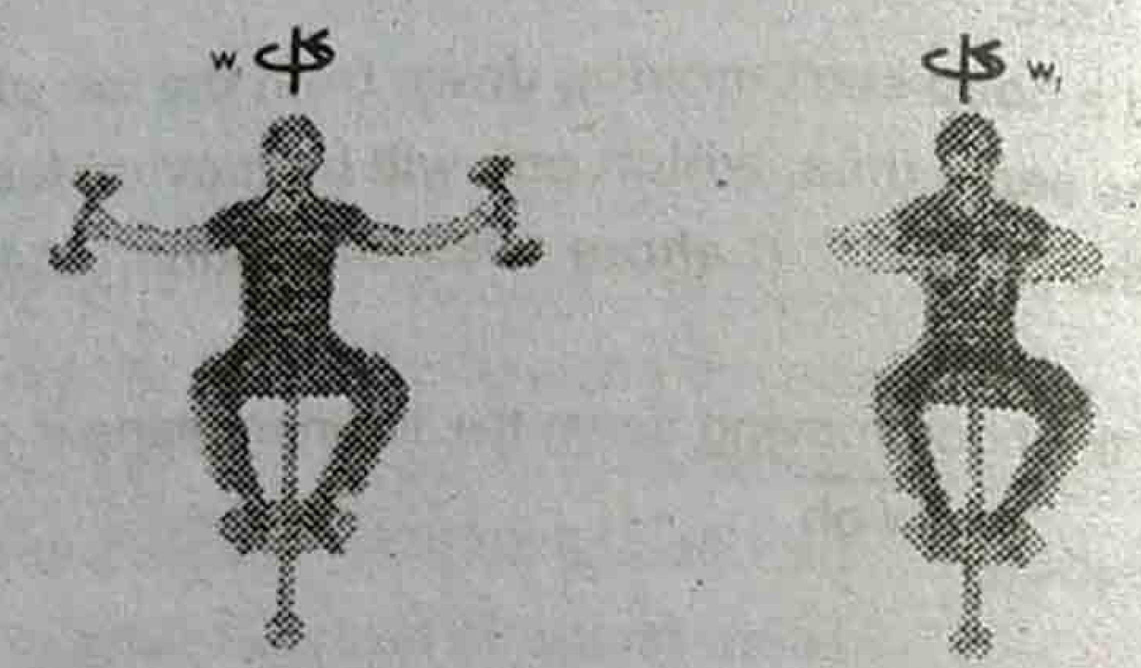
Ans:- A diver changes his body positions to spin (گھمانا) himself faster, so that he may be able to take extra somersaults (قلابازیاں).
For this purpose, when the diver lifts off (اوپر اٹھنا) the diving board, his legs and arms are fully extended (پھیلانا) in order to (کی غرض سے) have a large moment of inertia I_1 ($= mr_1^2$) about an axis. Thus his angular veocity `$\omega_1$' decreases. When he pulls his legs and arms into closed tuck position(اکٹھا سکیڑنا), his moment of inertia is reduced to a new value $I_2$ ( $= mr_2^2$ ). So the value of his angular velocity `ω_2' increases.
As the angular momentum is conserved, so

$$I_1\omega_1 = I_2\,\omega_2 = \text{constant}$$

Hence, the diver spins faster when moment of inertia becomes smaller and angular velocity increases to conserve angular momentum. In this way, he can make more somersaults.

NOTE:- When moment of inertia I_1 increases, it means that value of r_1 increases in the formula $I_1 = mr_1^2$, and r_2 decreases due to closed tuck porition in second case. ($I_2 = mr_2^2$)

Q.5.12. A student holds (تھامے) two dumb - bells with out-stretched arms while sitting on a turntable. He is given a push until he is rotating a certain angular velocity. The student then pulls the dumb-bell to wards his chest (Fig. 5.24). What will be the effect on rate of rotation?



(a) (b)

Fig. 5.25

Ans;- This is an application of law of conservation of momentum i.e.

$$I_1\omega_1 = I_2\omega_2 = \text{constant}$$

The angular momentum \`L' (=Iω) of the student lies along vertical axis. In the position of out - stretched arms, the moment of inertia of the student is $I_1$ and his angular velocity is \`ω_1'. According to the law of conservation of momentum, their product I_1,ω_1 remains constant. When he pulls the dumb - bells and arms closer to his chest, his rotational moment of inertia \`$I_2$' decreases because he brings the mass (dumb - ball) close to the axis of rotation. As $I_2 = mr_2^2$, when $r_2$ decreases then moment of inertia also decreases, but angular velocity \`ω^2' of the student increases to keep the product I_2w2 constant. Since the angular velocity in this case is greater than the initial value of angular velocity, so the student will rotate faster in the last case.

Hence, when he pulls his arms, his rate of rotation increases.

Q.5.13 Explain how many minimum number of geo-stationary satellites are required for global coverage of TV transmission.

Ans; The minimum number of geo-stationary satellites required for global coverage of T.V. transmission is 3 (three). Since one geo- stationary satellite covers 120o of longitude (طول بلد), so for the whole populated Earth's surface there must be 360o of longitude. Hence, the whole of Earth's surface for global coverage of T.V. transmission can be covered by three correctly positioned geo- stationary satellites.

NUMERICAL PROBLEMS WITH SOLUTIONS

P.5.1. A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a daimeter of 2.50 m at the moon, how small must divergence (پھیلاؤ) angle be for the beam? The distance of Moon from the Earth is 3.8 ×10^8m.

SOLUTION;-

DATA:-

Diameter of beam = length of arc = S = 2.50 m

Distance of Moon from the Eath = Radius of circular arc = r = 3.8 × 10^8 m

TO FIND:-

Divergence angle = θ = ?

FORMULA:-

$$S = r\theta$$

CALCULATIONS:-

Using the formula

$$S = r\theta$$

or $$\theta = S/r$$

Putting the values, we get

$$\theta = \frac{2.50}{3.8\ 10^8} = 6.6\times10^{-9} \text{ radians}$$

Hence, $\boxed{\theta = 6.6\times10^{-9}\text{ rad}}$ Ans.

RESULT:- The value of divergence of angle of beam = 6.6 × 10^{-9} radians.

P.5.2. A gramophone record turntable from rest to an angular velocity of 45.0 rev min^{-1} in 1.60s. What is its average angular velocity?

SOLUTION:-

DATA:-

Initial angular velocity = $\omega_i = 0$

final angular velocity = ω_f = 45.0 rev min^{-1}

$$= \frac{45.0\times2\pi}{60} = 1.5\pi \text{ rads}^{-1} \qquad (\text{rev} = 2\pi)$$

Time = t = 1.60 s

TO FIND:-

Average angular acceleration = α= ?

FORMULA:-

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

CALCULATIONS:-

Using the formula

$$\alpha = \frac{\omega_f - \omega_f}{t}$$

Putting the values, we get

$$\alpha = \frac{1.5\pi - 0}{1.60} = 2.95 \text{ rads}^{-2}$$

or $\boxed{\alpha = 2.95 \text{ rad s}^{-2}}$ **Ans.**

RESULT:- Average angular acceleration of gramophone record turntable is 2.95 rads^{-2}

P.5.3. **A body of moment of inertia $I = 0.80 \text{ kgm}^2$ about a fixed axis, rotates with a constant angular velocity of 100 rads^{-1}. Calculate the angular momentum `L' and the torque to sustain (قائم رکھنا) this motion.**

SOLUTION:-

DATA:-

Moment of inertia = $I = 0.80 \text{ Kgm}^2$

Angular velocity = $\omega = 100 \text{ rads}^{-1}$

TO FIND

Angular momentum = L = ?

Torque = τ = ?

FORMULA:-

$$L = I\omega$$

CALCULATIONS:-

Angular Momenum:-

Using the formula

$$L = I\omega$$

Putting the values, we have

$$L = 0.80 \times 100$$
$$= 80 \text{ kgm}^2\text{s}^{-1}$$

or $$L = 80 \text{ kgm}^2\text{s}^{-1} \times \frac{\text{s}^{-1}}{\text{s}^{-1}}$$

$$= 80 \text{ kgm}^2\text{s}^{-2}/\text{s}^{-1} \qquad (\because 1\text{kg m}^2\text{s}^{-2} = 1\text{J})$$

$\therefore$ $\boxed{L = 80 \text{ Js}}$ **Ans.**

TORQUE:- Torque can be found by using the relation

$$\tau = I\alpha$$

Since the motion has constant angular velocity, therefore angular acceleration is zero.

Thus,

$$\tau = I \times 0 = 0$$

$\therefore$ $\boxed{\tau = 0}$

RESULT:- The angular momentum of the body is 80 Js and torque to sustain the motion is zero.

P.5.4. **Consider the rotating cylinder shown in fig 5.26. Suppose that M=5.0 kg, F = 0.60 N and r = 0.20m. Calculate (a) Torque t acting on the cylinder (b) The angular acceleration `a' of the cylinder.**

(Moment of inertia of cylinder = 1/2 MR^2)

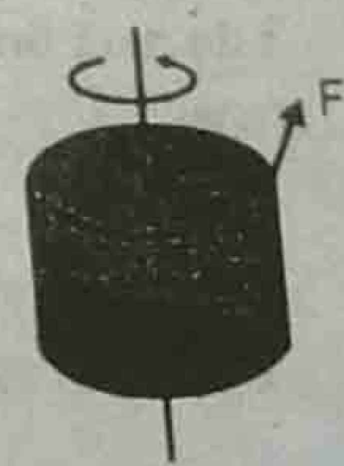


Fig. 5.26

SOLUTION:-

DATA:-

Mass of cylinder = M = 5.0 kg

Force acting on cylinder = F = 0.60 N

Distance = r = 0.20 m

TO FIND:-

(a) Torque acting on cylinder = t = ?

(b) Angular acceleration of cylinder = a = ?

FORMULA:-

(a) $\tau = rF\sin\theta$

(b) $\tau = I\alpha$

CALCULATIONS:-

(a) Using the formula of torque

$$\tau = rF\sin\theta$$

r and F are perpendicular to each other.

$$\tau = rF\sin 90$$

putting the values, we get $(\sin 90^\circ = 1)$

$$\tau = 0.20 \times 0.60 \times 1$$

or $\boxed{\tau = 0.12 \text{ Nm}}$ Ans.

(b) Now using the formula

$$\tau = I\alpha \quad \ldots\ldots\ldots\ldots$$

where I is the moment of inertia of cylinder which is equal to $\frac{1}{2}Mr^2$

Therefore,

$$\tau = \frac{1}{2} Mr^2\alpha$$

Putting the values, we have

$$\tau = \frac{1}{2} \times 5.0 \times (0.20)^2 \alpha$$

$$0.12 = 0.1\ \alpha$$

or $\alpha = 0.12/0.1 = 1.2\ \text{rads}^{-2}$

$\therefore$ $\alpha = 1.2\ \text{rds}^{-2}$ **Ans.**

RESULT:- The torque acting on the cylinder is 0.12 Nm and its angular acceleration is 1.2 rad s^{-2}

P.5.5. Calculate the angular momentum of a star of mass 2.0×10^{30} kg and radius 7.0×10^5 km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

SOLUTION:-

DATA:-

Mass of a star = M = 2.0×10^{30}

Radius of star = R = 7.0×10^5 km

= 7.0×10^8 m (..(km = 10^3m)

Time period of star = T = 20 days

=20 × 24×60×60s

= 1728000s

TO FIND:-

Angular momentum of star = L =?

kinetic energy = K. E. = ?

FORMULA:-

(i) $L = I\omega$

(ii) $K.E. = \frac{1}{2} I\omega^2$

CALCULATIONS:-

Using the formula

$$L = I\omega$$

where $I = \frac{2}{5} MR^2$ for a sphere

and $\omega = \frac{2\pi}{T}$

Thus, $$L = \frac{2}{5} MR^2 \times \frac{2\pi}{T}$$ ($\pi = 3.14$)

Putting the values, we have

$$L = \frac{2 \times 2\times10^{30} \times (7\times10^8)^2 \times 2 \times 3.14}{5 \times 1728000}$$

$$= \frac{2\times2\times7\times7\times2\times3.14 \times10^{46}}{5\times 1728\times10^3}$$

$$= 0.142462963\times 10^{43}$$

$$L = 1.4 \times10^{42}\ \text{kgms}^{-1} \times s^{-1}/s^{-1}$$ (kgms^{-2} = J)

$\therefore$ $\boxed{L = 1.4 \times 10^{42}\ \text{Js}}$ **Ans.**

(b) Using the formula of kionetic energy $K.E. = \frac{1}{2} I\omega^2$

Putting the values of I and w , we have

$$K..E. = \frac{1}{2} \times \frac{2}{5} MR^2 \times (2\pi/T)^2$$

or $$K..E. = \frac{1}{2} \times \frac{2}{5} \times 2 \times10^{30} (7\times10^8)^2 (2\pi/20\times24\times3600)^2$$

$$= \frac{1}{5} \times 2\times 10^{30} \times 49 \times 10^{16} \times (1.3 \times 10^{-11})$$

$$= 25.48 \times 10^{35}\ J$$

$$= 25 \times 10^{35} \times 10/10$$

$\boxed{K.E. = 2.5 \times10^{36}\ J}$ **Ans.**

RESULT:- (a) Angular momentum of star is 1.4×10^{42} Js and (b) K.E. of star is 2.5×10^{36} J

P.5.6 A 1000 kg car traveling with a speed of 144 kmh^{-1} rounds a curve of radius 100m. Find the necessary centripetal force.

SOLUTION:-

DATA:-

Mass of the car = M = 1000 kg

Speed of car = v = 144 kmh^{-1}

$$= \frac{144 \times 1000}{60 \times 60} = 40\ ms^{-1}$$

Radius of the curve = r = 100m

TO FIND:-

Centripetal force = F = ?

FORMULA:-

$$F_c = \frac{mv^2}{r}$$

CALCULATIONS:-

using the formula

$F_c = mv^2/r$ and putting the values, we get

$$F_c = \frac{1000\times(40)^2}{100} = 1.60\times10^4\ N$$

Hence $F_c = 1.60\times10^4\ N$

RESULT:- Centripetal force of car is $1.60\times10^4\ N$

P.5.7. What is the least speed at which an aeroplane can execute a vertical loop of 1.0 km radius so that there will be no tendency (رجحان) for the pilot to fall down at the highest point?

SOLUTION:-

DATA:-

Radius of loop = r = 1.0 km = 1000 m

Acceleration due to gravity = g = 9.8 ms^{-2}

TO FIND;-

Speed of aeroplane = v = ?

FORMULA:-

$$a_c = g = v^2/r$$

CALCULATIONS:-

When an aeroplane executes a circular loop, the centripetal acceleration is supplied by gravity and we have

$$a_c = g = v^2/r$$

or $$v^2 = rg$$

$$v = \sqrt{rg}$$

Putting the values, we get

$$v = 1000\times9.8$$

or $\boxed{v = 99\ ms^{-1}}$ **Ans.**

Result:- The least speed at which an aeroplane can execute a vertical loop is $99ms^{-1}$

P.5.8. The Moon orbits the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (In this case, treat the Moon as a particle orbiting the Earth). Distance between the Earth and the Moon is 3.85×10^8m. Radius of the Moon is 1.74×10^6 m.

SOLUTION:-

DATA:-

Distance between Earth and Moon = r = 3.85×10^8 m

Radius of Moon = $R_m = 1.74\times10^6$ m

TO FIND

Ratio of spin and orbital angular momentum = L_s/L_o = ?

FORMULA:-

$$L_s = I\omega$$

$$L_o = Mr^2\omega$$

CALCULATIONS:-

The spin angular momentum of the Moon about its axis is

$$L_s = I\omega$$

where $I = \frac{2}{5}MR^2_m$

Thus,

$$L_s = \frac{2}{5}MR^2_m\omega \quad\text{.......................... (1)}$$

The orbital angular momentum is given by

$$L_o = Mr^2\omega \quad\text{.......................... (2)}$$

where angular speed ω in both cases is the same and also the time to complete one rotation around Earth and one rotation around its axis is the same.

Dividing equation (1) by equation (2) we get

$$\frac{L_s}{L_o} = \frac{2\ MR^2_m\omega}{5\ Mr^2\omega} = \frac{2R^2_m}{5r^2} \quad\text{............... (3)}$$

Putting the values of R_m and r in equ (3) we get

$$\frac{L_s}{L_o} = \frac{2\times(1.74\times10^6)^2}{5\times(3.85\times10^8)^2}$$

$$= \frac{2\times3.03\times10^{12}}{5\times1.48\times10^{17}} = \frac{6.06\times10^{12}}{7.4\times10^{17}}$$

$$= 8.2\times10^{-6}$$

Henec, $\boxed{L_s/L_o = 8.2\times10^{-6}}$ **Ans.**

RESULT:- The ratio of the spin angular momentum to the orbital angular momentum is 8.2×10^{-6}.

P.5.9. The Earth rotates on its axis once a day. Suppose, by some process the Earth contracts so that the radius is only half as large as at present, how fast will it be rotating them? (Moment of inertia of sphere $I = 2/5\ MR^2$)

SOLUTION:-

DATA:-

Time period = T_1 = 24 hours

Moment of inertia of sphere = $I_1 = \frac{2}{5} MR_1^2$

Radius of earth $R_2 = \frac{1}{2} R_1$

TO FIND

How fast will the Earth be rotatating i.e T_2 = ?

FORMULA:-

$I_1\omega_1 = I_2\omega_2$

CALCULATIONS:-

According to the law of conservation of momentum

$$I_1\omega_1 = I_2\omega_2$$

or $$\frac{\omega_1}{\omega_2} = \frac{I_2}{I_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For sphere, $I_1 = 2/5\, MR_1^2$

and $$I_2 = \frac{2}{5} MR_2^2 = \frac{2}{5} M \frac{(R_1)^2}{(2)^2}$$

As we have

$$\omega_1 = \frac{2\pi}{T_1} \text{ and } \omega_2 = \frac{2\pi}{T_2}$$

Putting these values in equation ,(1) we get

$$\frac{2\pi/T_1}{2\pi/T_2} = \frac{2/5\, M (R_1/2)^2}{2/5\, MR_1^2}$$

or $T_2/T_1 = 1/4$

or $T_2 = \frac{T_1}{4}$

or As we know that

T_1 = 1 day = 24 hours

Therefore, T_2 = 24/4 = 6 hours

Hence $\boxed{T_2 = 6 \text{ hours}}$ **Ans.**

RESULT:- The Earth would complete its rotation in 6 hours.

P5.10.-What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take the mass of Earth as 6.0×10^{24} kg and its radius as 6400 km)

SOLUTION:-

DATA:-

Mass of Earth = M = 6.0×10^{24} kg

Radius of Earth = R = 6400km

Height of circular orbit = h = 900 km

TO FIND:-

Orbital speed = v = ?

FORMULA:-

$v = \sqrt{GM/r}$

CALCULATIONS:-

Using the formula, $v = \sqrt{GM/r}$

where 'r' is the total distance from the centre of Earth.

Here $r = R + r_e$

Thus, r = 6400+900 = 73 00 km = 7300×10^3m

Putting the values, we get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{7300 \times 10^3}}$$

$$= \sqrt{54.8 \times 10^6}$$

or $v = 7.4 \times 10^3$m

Hence $\boxed{v = 7.4 \text{ km}^{-1}}$ ($10m^3$ = 1km)

RESULT:- The orbiting speed to launch a satellite in a circular orbit is $7.4\ kms^{-1}$.

IMPORTANT BOARD QUESTIONS

Q.1. (a) Explain circular motion, angular displacement. Give units.
(Sargodha board 2002, Gujranwala 1998, RWP. 2003)

(b) Define radian Prove, $S = r\theta$
(D.G. Khan s, 2001, Bahawalpur 2001, gujranwala board 1998, Sargodha 2001)

(d) Derive a relation between a radian and degree (or one radian = 57.3)
(Multant bord 2001, Gujranwala 1998)

Q.2. (a) What are average and instantaneous angular velocities.
(Bahawalpur board 2002, Gujranwala 1989)

(b) Define average angular acceleration and instantanous angular acceleration
(Gujranwala board s, 1989, Lahore 1995, 2003, RWP. 2003)

Q.3. Define average angular acceleration and instantanous angular acceleration.

OR

Prove $\therefore v = r\omega$
(Faisalabad board 2002, Gujranwala 2002, Multan 2001, D. G. Khan s 2001, 2002 Bahwalpur 2002, Sargodha 1997, RWP. 2003)

(b) Derive the relation between linear acceleration and angular acceleration

OR

Prove $a = r\alpha$
(Multan board 2001, D. G. Khan s , 2001, D. G. Khan s 2001, 2002, Sargodha 1997, Lahore 2000S, Gujranwala 1999, RWP. 2003)

Q.4. (a) What is meant by centripetal forc4e and centripetal acceleration?
(Lahore board 2001, Gujranwala 2002, Bahawalpur 2000, Sargodha 1998, Faisalabad 1997, Multan 2003, RWP. 2003)

(b) Berive expressions for

(i) Centripetal acceleration, a V^2/r (or $a = w\omega^2 r$)
(Lahore board 2001, Gujranwala 2002, Bahawalpur 2000, Rawalpindi 2001 s., Faisalabad 1997, Sargodha 1998, Multan 2003)

(ii) Centripetal force, $F = \frac{mv^2}{R} = m\omega^2 r$

(iii) What is the difference between centripetal and centrifugal (or reacting) forces (Bahwalpur board 2000, 2001, Gujranwala 1998)

(iv) What is radian? Show one radian $= \frac{180}{\pi}$
(Multant board 2001, S 2002, Gujraneal 1999)

Q.5. (a) What is moment of inertia? Find an expression for inertia of mass 'm' rotating about the point 'O' (axis of rotation)
(Lahore board 1984, Guj 1985, 88, D.G. Khan 1990)

(b) Find the expression for moment of inertia of a rigid body
(Lahore board 1984, 86, D. G. Khan 1990)

Q.6. (a) Explain angular momentum. Derive a relation between angular momentum and moment of inertia

(b) Also describe the difference between spin angular momentum and orbital angular momentum.
(Azad Kashmir 2002, Lahore 1984, 87, Sargodha 1995)

Q.7. State the law of conservation of angular momentum and illustrate it with example. Also describe its applications
(Bhawalpur 1996, Faisalabad 199, lahore 2000, S, Federal board 2000, 2001)

Q.8. (a) Define rotational kinetic energy. Show the kinetic energy of a rotating body is given by ½ Iω2, where down its practical use.
(Lahore board 1984, 1986, Gujranwala 1988, D. G. Khan 1991, Faisalabad 1991, 89, Sargodha 1986, 1989, Multan 1989,85)

(b) Find the rotational kinetic energies of dise and hoop. Also derive the relations for the velocities of a disc and hoop moving down and inclined plane.
(Rawalpindi board 1984, Gujranwala 1985, Lahore 1988, D. G. Khan 1990)

Q.9. What are artificial satellites? Find the expression for minimum velocity and period to put a satellite into the orbit.
(Lahore board 1983, Faisalabad 19985, Bhawalpur 1988)

Q.10. (a) What do you understand by real and apparent weight?

(b) An object is suspended in a lift or elevator by a string and spring balance. Find the apparent weight of the object in the following cases.

(i) When the lift is rest
(ii) When the lift moves upward with uniform acceleration 'a'
(iii) When the lift moves downward with uniform acceleration 'a'
(iv) When the lift is falling freely under gravity.

(D. G. Khan board 1999, 2002, Lahore 1989, Gujrawala 1997, Sargodha 1989, Faisalabad 1996)

Q.11. It is said that an astronaut (man) and other objects in the satellite orbiting the Earth are weightless. Explain this phenomenon "Weightlessness in satellites"
(lahore board supp. 2001, Multan 2000, Gujranwala 19987, Faisalabad 1990, Sargodha 1984, Federal board board 2002)

Q.12. What is orbital motion? Derive an expression for orbital velocity
(Faisalabad board 2002, Gujranwala 2000)

Q.13. Explain, what do you understand by "Artificial Gravity"? Derive an expression for frequency with which the spaceship (spacecraft) rotates to provide artificial gravity like Earth. ***(Rawalpindi 2003)***

Q.14. What are geo stationary satellites? Derive an expression for the orbital radius of geostationary satellite? ***(Sargodha 2003)***

Q.15. Write a note communication satellites.

Q.16. Discuss Newton's and Einstein's views of gravitation.